On Economic Inequality

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To Antara and Nandana

*with the hope that when they grow up
they will find less of it no matter
how they decide to measure it*

Preface

The idea of inequality is both very simple and very complex. At one level it is the simplest of all ideas and has moved people with an immediate appeal hardly matched by any other concept. At another level, however, it is an exceedingly complex notion which makes statements on inequality highly problematic,[1] and it has been, therefore, the subject of much research by philosophers, statisticians, political theorists, sociologists and economists. While this book is concerned with economic inequality only, the presentation reflects this duality. I have had to employ a fair number of technical concepts and use some mathematical operations, but the concepts have also been explained in non-technical terms and the mathematical results have been given intuitive explanation. It is hoped that the non-technical reader will not be put off by the formalities. The importance of the formal results lies ultimately in their relevance to normal communication and to things that people argue about and fight for.

While the technical and non-technical sections have not been put into separate compartments, it should be possible for someone not interested in technicalities to skip (or skim through) the formal sections and to go directly from the intuitive presentation of the axioms to the intuitive explanation of the results. The section headings used throughout the book should help the reader in this sorting out.

In many ways this book is a development of some ideas I studied in my *Collective Choice and Social Welfare*.[2] The framework of thought presented there I have tried to apply here to the specific field of economic inequality. The approaches to social evaluation that I rejected then, I reject more strongly now, and what I defended in that work, I have tried to develop

[1] See Bernard Williams, 'The Idea of Equality', in P. Laslett and W. G. Runciman, *Philosophy, Politics and Society*, Second Series, Blackwell, Oxford.

[2] Holden-Day, San Francisco, 1970, and Oliver & Boyd, Edinburgh, 1971, Mathematical Economics Texts, No. 5.

more fully in this one in the particular context of inequality. No apologies for that, but I ought to put my cards on the table.

I owe debts to many. While preparing the Radcliffe Lectures, I was working with Partha Dasgupta and David Starrett on a joint paper on the measurement of economic inequality.[3] I am grateful to them not only because I have incorporated into the lectures some results from our joint paper (in particular, Theorems 3.1 and 3.2), but also because I have learnt a great deal from them and I have used that knowledge quite freely.

The Radcliffe Lectures, which were delivered last May, were informally presented, and in the discussions that followed I have gained much. I should particularly mention the searching questions raised by David Epstein, John Muellbauer, Graham Pyatt and John Williamson. In revising the lectures for this book, I have expanded some sections, incorporating not merely those things that I could not put into the lectures because of shortage of time or because of stylistic limitations (footnotes sound nasty in a lecture), but also some additional bits which are essentially responses to the queries raised. I have also benefited from discussions following my lectures on related topics at Essex University (Economics Department Seminar, January 1972), Columbia University (Joint Seminar of Economics and Philosophy Departments, March 1972), Harvard University (Political Economy Lecture, March 1972), the Delhi School of Economics (Special Lectures, August 1972), and the Indian Statistical Institute (Research Seminars, August 1972). I am grateful to Tony Atkinson, Pranab Bardhan, Nikhiles Bhattacharya, Sanjit Bose, Terence Gorman, Peter Hammond, and Richard Layard, for helpful comments and criticisms. This is a long list, and there must have been others.

For astonishingly skilful typing against the heavy odds of my impossible writing, I am very grateful to Celia Turner and Luba Mumford.

[3] 'Notes on the Measurement of Inequality', forthcoming in the *Journal of Economic Theory*.

Finally I am most grateful to the University of Warwick, and in particular to Professor Graham Pyatt, for the honour of an invitation to deliver the Radcliffe Lectures for this year.

London School of Economics A.K.S.
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Welfare Economics, Utilitarianism, and Equity

'OF all human sciences the most useful and most imperfect appears to me to be that of mankind: and I will venture to say the single inscription on the Temple of Delphi[1] contained a precept more important and more difficult than is to be found in all the huge volumes that moralists have ever written.' Thus wrote Jean Jacques Rousseau in the Preface to his *A Dissertation on the Origin and Foundation of the Inequality of Mankind*, dedicated to the Republic of Geneva on the 12th of June 1754. While the essay, alas, failed to qualify for the prize of the Dijon Academy for which it was considered (and which his less rebellious earlier piece on 'arts and sciences' had received in 1750), the ideas contained in it did help to crystalize the demands that gripped the revolution of 1789.

The relation between inequality and rebellion is indeed a close one, and it runs both ways. That a perceived sense of inequity is a common ingredient of rebellion in societies is clear enough, but it is also important to recognize that the perception of inequity, and indeed the content of that elusive concept, depend substantially on possibilities of actual rebellion. The Athenian intellectuals discussing equality did not find it particularly obnoxious to leave out the slaves from the orbit of discourse, and one reason why they could do it was because they could get away with it. The concepts of equity and justice have changed remarkably over history, and as the intolerance of stratification and differentiation has grown, the

[1] The Delphic injunction, it may be recalled, was the somewhat severe advice: 'Know thyself!'

very concept of inequality has gone through radical transformation.

In these lectures I am concerned with *economic* inequality only, and that again in a specific context,[2] but I should argue that the historical nature of the notion of inequality is worth bearing in mind before going into an analysis of economic inequality as it is viewed by economists today. Ultimately the relevance of our ideas on this subject must be judged by their ability to relate to the economic and political preoccupations of our times.

Objective and normative features

The main focus of these lectures will be on the problem of the measurement of inequality of income distribution in aggregative terms, though I shall try to go into some of the policy issues, especially in the context of the socialist economy. On the question of the measurement of inequality, we might begin with a methodological point. The measures of inequality that have been proposed in the economic literature fall broadly into two categories. On the one hand there are measures that try to catch the extent of inequality in some *objective* sense, usually employing some statistical measure of relative variation of income,[3] and on the other there are indices that try to measure inequality in terms of some *normative* notion of social welfare so that a higher degree of inequality corresponds to a lower level of social welfare for a given total of income.[4] It is possible to argue that there are some advantages in taking the former approach, so that one can distinguish between (a) 'seeing' more or less inequality, and (b) 'valuing' it more or less in ethical terms. In the second approach inequality ceases to be an objective notion and the problem of measurement is enmeshed with that of ethical evaluation.

[2] In particular I shall be concerned primarily with the distribution of *income* and not directly with *wealth*.

[3] The usual measures include the variance, the coefficient of variation, the Gini coefficient of the Lorenz curve, and other formulae, which will be discussed in Chapter 2.

[4] For examples of the normative approach to the measurement of income distribution, see Dalton (1920), Champernowne (1952), Aigner and Heins (1967), Atkinson (1970), Tinbergen (1970), and Bentzel (1970).

This methodological point essentially reflects the dual nature of our conception of inequality. There is, obviously, an objective element in this notion; a fifty-fifty division of a cake between two persons is clearly more equal in some straightforward sense than giving all to one and none to the other. On the other hand, in some complex problems of comparing alternative income distributions among a large number of people, it becomes very difficult to speak of inequality in a purely objective way, and the measurement of the inequality level could be intractable without bringing in some ethical concepts.

Which of the two approaches it would be correct to pursue is not an easy question to answer, and the two approaches in terms of their practical use would not be all that different from each other. Even if we take inequality as an objective notion, our interest in its measurement must relate to our normative concern with it, and in judging the relative merits of different objective measures of inequality, it would indeed be relevant to introduce normative considerations. At the same time, even if we take a normative view of the measures of income inequality, this is not necessarily meant to catch the totality of our ethical evaluation. It would presumably aim to express one particular aspect of the normative comparison, and which particular aspect will depend on the objective features of the inequality problem. To say that 'x involves less inequality than y', even if meant to be a normative statement, will not imply an unqualified recommendation to choose x rather than y, but would presumably be combined with other considerations (e.g., those involving total income and such features) to arrive at an overall judgement.[5] In one way or another, usable measures of inequality must combine factual features with normative ones.

Types of measurement

A second methodological issue concerns the type of measurement that is being sought. Various degrees of measurement

[5] In terms of the classification of value judgements used in Sen (1967), inequality judgements are *non-compulsive evaluative* judgements.

are conceivable. The strictest type of measure is a ratio-scale like weight or height, in which it makes sense to say that one object weighs twice as much as another (and it does not matter whether we measure it in kilogrammes or pounds). A somewhat looser measure is that of an interval-scale, in which ratios make no sense but the ratios of differences do. The gap between 100° Centigrade and 90° Centigrade is recorded as twice that between 90°C and 85°C no matter whether we express these temperatures in Centigrade or in Fahrenheit (in which they correspond respectively to 212°F, 194°F and 185°F), but the ratio of the temperatures themselves will vary according to the scale chosen.

This interval-scale measure is usually referred to in utility theory as 'cardinal', and if a set of numbers x represents the utilities of different objects, a positive linear transformation of these numbers such as $y = a + bx$, with $b > 0$, can also be used.[6] A looser measure than this corresponds to what is called an 'ordinal' scale in utility theory, where any positive monotonic transformation will do as well, e.g., a set of numbers 1, 2, 3, 4 can be replaced by 100, 101, 179, 999, respectively, since the ranking of the numbers is all that matters.

A closely related measure to the 'ordinal' scale does not involve any numerical representation whatsoever, and just an ordering of all the alternatives is presented, e.g., a set of four alternatives, x_1, x_2, x_3 and x_4, may be ranked as x_3 highest, x_2 and x_1 next together and x_4 last. This kind of an ordering involves a ranking with two specific properties, viz., completeness and transitivity. Completeness requires that if we take any pair of alternatives then in terms of the ranking relation R, either xRy holds or yRx holds, or both. Interpreting R as the relation 'at least as good as', if xRy holds but not yRx then we can say that x is strictly better than y and indicate this as xPy; the case of yPx is exactly the opposite of this. If both xRy and yRx hold, we can declare x and y as 'indifferent' and refer to this as xIy. The property of transitivity demands that if we take any three alternatives x, y, z, and xRy and yRz both

[6] For example, if F is the temperature in the Fahrenheit scale and C that in the Centigrade scale, we have: $F = 32 + 1{\cdot}8C$.

hold, then so does xRz. It might appear that an ordering can be easily converted into an 'ordinal' numerical measure, and this is indeed so for a finite set of alternatives, but is not invariably possible for an infinite set.[7] In fact, an ordering is a weaker requirement than the existence of an ordinal numerical representation.

Quasi-orderings and inequality judgements

A still weaker measure would be a case where the ranking relation R is not necessarily complete, i.e., not all pairs are rankable vis-à-vis each other. If a relation like this is transitive but not necessarily complete, it is called a quasi-ordering. Another case also weaker than an ordering is one where the ranking relation is complete but not necessarily transitive, of which a special case is one where the strict preference is transitive but indifference is not so.[8]

Most statistical measures of the inequality level assume a high degree of measurement, usually a ratio-scale or at least an interval-scale. This is true not only of the so-called objective measures, but also of normative evaluation (see Chapter 2). It is, however, possible to argue that the implicit notion of inequality that we carry in our mind is, in fact, much less precise and may correspond to an incomplete quasi-ordering. We may not indeed be able to decide whether one distribution x is more or less unequal than another, but we may be able to compare some other pairs perfectly well. The notion of inequality has many aspects, and a coincidence of them may permit a clear ranking, but when these different aspects conflict an incomplete ranking may emerge. There are reasons to believe that our idea of inequality as a ranking relation may indeed be inherently incomplete. If so, to find a measure of inequality that involves a complete ordering may produce artificial problems, because a measure can hardly be more precise than the

[7] The problem arises from not necessarily having a sufficient stock of real numbers to give each alternative an appropriate number in special cases such as lexicographic orderings over a many-dimensional real space. On this see Debreu (1959), Chapter 4.

[8] See Fishburn (1970), Sen (1970), and Pattanaik (1971).

concept it represents. It will be argued in Chapter 3 that this might well account for some of the difficulties with the standard measures of inequality.

In this context it is perhaps worth saying that the historical connection between the notion of inequality and discontent—and more so rebellion—suggests that the need is for a measure that comes into its own with sharp contrasts, even though it may not provide a scale sensitive enough to order finely distinguished distributions. The unfortunate fact is that in putting up a scale of measurement or ranking, the economist's and the statistician's inclination is to look for an ordering complete in all respects, so that the translation of the notion of inequality from the sphere of political debate, which gives the notion its importance, to the sphere of well-defined economic representation may tend to confuse the mathematical properties of the underlying concept. Indeed inequality measurement is by no means the only field of economic analysis in which the predisposition towards a complete ordering has proved to be a major liability.

Non-conflict economics and Pareto optimality

How much guidance—it is reasonable to ask—can we expect to get from modern welfare economics in analysing problems of inequality? The answer, alas, is: not a great deal. Much of modern welfare economics is concerned with precisely that set of questions which avoid judgements on income distribution altogether. The concentration seems to be on issues that involve no conflict between different individuals (or groups, or classes), and for someone interested in inequality this can hardly make the air electric with expectations.

The so-called 'basic' theorem of welfare economics is concerned with the relation between competitive equilibria and Pareto optimality.[9] The concept of Pareto optimality was evolved precisely to cut out the need for distributional judgements. A change implies a Pareto-improvement if it

[9] For the relevant theorems with proofs see Debreu (1959) and Arrow and Hahn (1972), and for an illuminating informal discussion see Koopmans (1957).

makes no one worse off and someone better off. A situation is Pareto optimal if there exists no other attainable situation such that a move to it would be a Pareto-improvement. That is, Pareto optimality only guarantees that no change is possible such that someone would become better off without making anyone worse off. If the lot of the poor cannot be made any better without cutting into the affluence of the rich, the situation would be Pareto optimal despite the disparity between the rich and the poor.

Suppose we are considering the division of a cake. Assuming that each person prefers to have more of the cake rather than less of it, every possible distribution will be Pareto optimal, because any change that makes someone better off is going to make someone else worse off. Since the only issue in this problem is that of distribution, Pareto optimality has no cutting power at all. The almost single-minded concern of modern welfare economics with Pareto optimality does not make that engaging branch of study particularly suitable for investigating problems of inequality.

Social welfare functions

At a more general level, however, there has been quite a bit of discussion in recent years on distributional judgements going beyond Pareto optimality, and indeed the famous Bergson–Samuelson social welfare function was partly motivated by the recognition that policy decisions in economics would require the economist to go beyond Pareto optimality. In its most general form the Bergson–Samuelson social welfare function is any ordering of the set of alternative social states. If X is the set of social states, then a Bergson–Samuelson social welfare function is an ordering R defined over the entire X. In numerical terms it was conceived of as a functional relation W that specifies a welfare value $W(x)$ for each social state x belonging to the set X. The measure of W has been usually taken to be 'ordinal'.

While this is the most general conception of the social welfare function, something more has to be said about the nature of the function $W(x)$ to get some results of practical importance out of this concept. A favourite assumption has been that the social welfare function is 'individualistic' in the sense of making social welfare W a function of individual utilities, i.e., $W(x) = F(U_1(x), \ldots, U_n(x))$, where U_i stands for the utility function of individual i, for $i = 1, \ldots, n$.[10] Further, assuming that W increases with any U_i given the set of utilities of all other individuals, Pareto optimality can be built into the exercise of maximizing W. But the main object of the social welfare function is to take us *beyond* this limited concept by ranking all the Pareto optimal states vis-à-vis each other. The distributional judgements would then depend on the precise social welfare function chosen.

While the conception of a function such as F permits the use of cardinal utilities of individuals as well as of interpersonal comparisons, orthodox welfare economics has been somewhat neurotic about avoiding both these activities. Much of the concentration has, therefore, been on arriving at social welfare, or at any rate at an ordering R of the set of social states X, based exclusively on the set of individual orderings of X. Representing the ordering of individual i as R_i, this line of thinking leads to the search for a functional relation $R = f(R_1, \ldots, R_n)$.

A natural question to ask in this context is whether certain general conditions can be imposed on the relation between the set of individual preferences and the social ordering. In a justly celebrated theorem, Arrow (1951) has shown that a set of extremely mild-looking restrictions eliminate the possibility of having any such functional relation f whatsoever. I do not intend here to go into Arrow's 'impossibility theorem', which

[10] See, for example, Bergson (1938), Lange (1942) and Samuelson (1947). Lange, however, seems to have thought that even if social welfare were based '*directly* [on] the distribution of commodities or incomes between the individuals, without reference to the individuals' utilities', social welfare could still 'be expressed in the form of a *scaler function of the vector u*, i.e., W(u)' (p. 30). While it is true that for any distribution of commodities or incomes there would be one and only one vector u and one and only one W, we could still have two distributions leading to the same vector u but to two different values of W, so that in this case W could not really be viewed as a function of u.

has produced much awe, some belligerence, and an astounding amount of specialized energy devoted to finding an escape route from the dilemma. Instead I wish to present a theorem which does not rule out all functional relations f but only those that express any distributional judgements whatsoever, thereby ruling out any meaningful discussion of inequality within the logical framework of this model. The object of presenting and discussing this result is to clarify a basic weakness of the approach in handling problems of distribution and inequality.

A result concerning distributional judgements

Given Arrow's 'impossibility' result, it is clear that the system needs some give. This we provide by relaxing the requirement that social preference R be an ordering, in particular the requirement that R be 'transitive' (i.e., that xRy and yRz should imply xRz). Instead we demand only that the strict preference relation P be transitive (without indifference being necessarily transitive). We continue to require that R should be 'complete', i.e., either x is regarded as at least as good as y, or y regarded as at least as good as x (or both, in which case indifference holds), and of course that R should be 'reflexive', which is the entirely reasonable demand that x be regarded as at least as good as itself. Altogether we impose five conditions on the relation f between individual preference orderings and social preference relation R.

Condition Q (*Quasi-transitive Social Preference*): The social preference R must be reflexive, complete and quasi-transitive, i.e., the range of f must be confined to preference relations R that are reflexive and complete and which involve a transitive strict preference relation P.

Condition U (*Unrestricted Domain*): Any logically possible combination of individual preference orderings can be admitted.

Condition I (*Independence of Irrelevant Alternatives*): Social preference R over any pair x, y depends only on individual preferences over x, y.

Condition P (*Pareto Rule*): For any pair x, y if all individuals find x to be at least as good as y and some individual finds x

to be strictly better than y, then x is socially strictly preferred to y; and if all individuals are indifferent between x and y, then so is society.

Condition A (*Anonymity*): A permutation of individual orderings over the individuals keeps the social preference unchanged.

The first condition permits systematic social choice. The second permits individuals to have any preference pattern. The third establishes a relation between individual and social preferences that can be viewed pair by pair. The fourth is simply the familiar Pareto rule. The last condition—originally introduced by May (1952) in the context of the simple majority rule—requires that no special importance should be attached to who in particular holds which preference, all that matters being the combination of preferences that are held (no matter who holds what). These conditions may look reasonable enough, but together they rule out distributional judgements *in toto*.[11]

Theorem 1.1

The only functional relation f satisfying Conditions Q, U, I, P, and A must make all Pareto-incomparable states socially indifferent.

There are various alternative ways of proving this theorem, and I give here the sketch of a proof which I have spelt out elsewhere.[12] Define a person k as 'semidecisive'[13] if his preferring any x to any y implies that socially x is regarded as at least as good as y. He is 'almost semidecisive' if xRy holds whenever he prefers x to y and furthermore everybody else prefers y to x. By using Conditions Q, U, P, and I, it can be shown that if a person is almost semidecisive over some ordered

[11] This theorem was presented in a slightly different version in Sen (1970) as Theorem 5*3.

[12] Sen (1970), pp. 75–7.

[13] This is a weakening of Arrow's (1963) definition of a set of individuals being 'decisive'.

pair (x, y) then he must be semidecisive over every ordered pair. I shall not spell out the entire argument here, but only demonstrate how the argument works. Assume that everyone other than k prefers y to x and also y to z, and let person k prefer x to y and y to z. By the Pareto rule, yPz. If we now assume that zPx, by quasi-transitivity (Condition Q) we would end up getting yPx; but since k is almost semidecisive over (x, y) clearly xRy. So zPx is false, and since R must be complete, xRz holds. By Condition I this must depend on individual preferences only over (x, z). Since only k's preference over (x, z) has been specified, k must be semidecisive over (x, z). Proceeding this way it can be shown that k would be semidecisive over every ordered pair in the set of social alternatives S.[14]

Next, a set V of individuals is 'almost decisive' over a pair (x, y) if as a result of everyone in V preferring x to y, and everyone not in V preferring y to x, the social ranking is xPy. The group of all individuals is, of course, an almost decisive set by virtue of the Pareto principle. Let the smallest almost decisive set for any pair in S be V^*, and let V^* be almost decisive over (x, y). Partition V^* into V_1^*, consisting of one person, and V_2^* the rest. The rest of the people not in V^* form set N. Let everyone in V_1^* prefer x to y and y to z, everyone in V_2^* prefer z to x and x to y, and everyone in N prefer y to z and z to x. Since V^* is almost decisive, clearly xPy. If we take zPy, this would make V_2^* almost decisive set, which is impossible since V^* was the *smallest* almost decisive set. Hence yRz. If we now take zPx, then by quasi-transitivity we would get zPy and end up in a contradiction. Hence xRz. But then the solitary man in V_1^* is almost semidecisive over (x, z), and therefore must be semidecisive over every ordered pair of alternatives.

So far Condition A (anonymity) has not been used at all. Using that we see that *everyone* must be semidecisive over *every* ordered pair. But then for x to be socially preferred to y, it is necessary that no one regards y to be better than x. That is,

[14] See the proof of Lemma 5*f in Sen (1970).

we need then that everyone regards y as being at least as good as x, which means that either (i) everyone is indifferent between x and y, or (ii) someone prefers x to y and everyone regards x to be at least as good as y. By Condition P, (i) implies that x and y are socially indifferent and (ii) implies that x is socially preferred to y. So x is socially preferred to y if and only if x is Pareto-superior to y. This means that if x is not Pareto-superior to y, then y is socially at least as good as x. And if x and y are Pareto-incomparable, then each is socially as good as the other and they must be socially indifferent.

Interpretation of Theorem 1.1

Theorem 1.1 makes Pareto comparisons the only basis of social choice. Since Pareto optimal points by definition are either Pareto-indifferent or Pareto-incomparable, they must all be declared socially indifferent. Even if one person prefers one state to another—however mildly—and all others have the opposite preference, the two states must still be declared to be *equally* good from the social point of view given the axioms of Theorem 1.1. We are back to a situation where judgements on inequality are not permitted and Pareto optimality is both necessary *and* sufficient for overall social optimality. Anyone wishing to make distributional judgements must reject something or other in the framework of Theorem 1.1.

Which of the five conditions is guilty? I would argue that the real trouble lies in the very conception of a social welfare function, which makes social preference dependent on individual orderings only, using neither valuations of intensities of preference, nor interpersonal comparisons of welfare. Avoiding interpersonal comparisons has been the dominant tradition in economics since the depression of the nineteen-thirties, for reasons that must have been—I suspect—unconnected with the depression itself, since the celebrated lambasting of interpersonal comparisons by Robbins (1932), (1938), and others, which started it all, could have hardly been inspired by the sight of obvious human misery. Be that as it may, the attempt

to handle social choice without using interpersonal comparability or cardinality had the natural consequence of the social welfare function being defined on the set of individual orderings. And this is precisely what makes this framework so remarkably unsuited to the analysis of distributional questions. The conditions used in Theorem 1.1 simply precipitate this fundamental weakness.

The point can be illustrated in terms of the exercise of dividing a cake of volume 100 between two persons 1 and 2, with $y_1 + y_2 = 100$, assuming that each prefers more to less. Armed only with individual orderings we know that person 1 prefers a 50–50 division to a 0–100 division, while person 2 prefers the latter. Now comparing the 50–50 division with a 49–51 division, we still have exactly the same ranking on the part of both individuals. We cannot say that the preferences were much sharper in the first case than in the second, since cardinality of individual utilities is not admitted; and this, combined with the ruling out of interpersonal comparisons, kills twice over any prospect of being able to make a statement of the kind that the gain of person 1 in going from 0 to 50 may be larger than the loss of person 2 in coming down from 100 to 50, or even from 51 to 50. The ruling out of interpersonal comparisons even eliminates the possibility of our being able to say that person 2 is better off than person 1 under a 0–100 division. In fact all the characteristics of individual welfare levels in the distribution problem are precisely left out of account in this framework, and it is no wonder that a set of fine-looking conditions can complete the kill and eliminate distributional judgements altogether. Thus Theorem 1.1.

Interpersonal comparisons

The crucial question really concerns interpersonal comparability, since cardinality alone—it is easy to check—will not help us much. With cardinality we can compare each person's gains and losses with alternative values of his own gains and losses, but distributional judgements would seem to demand some ideas of the relative gains and losses of different persons and also of their relative levels of welfare. Indeed Arrow's 'impossibility theorem', which I referred to earlier, remains virtually intact even when cardinality is introduced in the absence of interpersonal comparability, as has been shown.[15] Theorem 1.1 has the same characteristic.

It seems reasonable, therefore, to argue that if the approach of social welfare functions is to give us any substantial help in measuring inequality, or in evaluating alternative measures of inequality, then the framework must be broadened to include interpersonal comparisons of welfare. The question will be asked at this stage whether such comparisons are at all legitimate, and if so in what sense. Despite the widespread allergy to interpersonal comparisons among professional economists, it is I think fair to say that such comparisons can be given a precisely defined meaning. In fact, various alternative frameworks are possible.[16] One in particular will be pursued here.[17]

If I say 'I would prefer to be person A rather than person B in this situation', I am indulging in an interpersonal comparison. While we do not really have the opportunity (or perhaps the misfortune, as the case may be) of in fact becoming A or B, we can think quite systematically about such a choice, and indeed we seem to make such comparisons frequently.

Representing (x, i) as being individual i (with his tastes and mental qualities as well) in social state x, a preference relation $\tilde{R}$ defined over all such pairs provides an 'ordinal' structure of interpersonal comparisons.[18] To obtain interpersonally comparable cardinal welfare levels, one would have to go beyond such a ranking $\tilde{R}$ and introduce additional features for the sake of cardinalization.[19] The numerical representation of $\tilde{R}$ will be unique only up to an increasing monotonic transformation if the measure is ordinal, and unique up to a positive linear transformation if it is cardinal. For any individual welfare function chosen $U(x, i)$ will stand for the welfare level of being person i in state x. While bearing in mind this general framework for interpersonal comparisons, we shall, however, represent this as $U_i(x)$ and think of this as our view of the welfare function of individual i. If the framework of complete interpersonal comparability is used, we must also specify that if any particular transformation of U_i is done for any individual i, then a corresponding transformation would have to be done to everyone else's welfare function as well. For example, given an accepted configuration of welfare functions of the different individuals, if one person's welfare function is doubled, then so should be the welfare function of everyone else as well. While the precise set of welfare functions chosen remains arbitrary, which is unavoidable given the fact that welfare has no natural 'unit' or 'origin', arbitrary *relative* variations are not permitted in the framework of 'full comparability'.[20]

[15] Theorem 8*2 in Sen (1970).

[16] See Vickrey (1945), Fleming (1952) and Harsanyi (1956). On the philosophical side, see in particular Kant (1788), Sidgwick (1874), Hare (1952), Rawls (1958), (1971) and Suppes (1966).

[17] Cf. Sen (1970) and Pattanaik (1971).

[18] Formally, $\tilde{R}$ is a ranking of the Cartesian product of X (the set of social states) and J (the set of individuals).

[19] For a discussion of the axiomatic approach to cardinalization, see Fishburn (1970).

Utilitarianism

Once the information content of individual preferences has been broadened to include interpersonally comparable cardinal welfare functions, many methods of social judgement become available. The most widely used approach is that of utilitarianism in which the sum of the individual utilities is taken as the measure of social welfare, and alternative social states are ordered in terms of the value of the sum of individual utilities. Pioneered by Bentham (1789), this approach has been widely used in economics for social judgements, notably by Marshall (1890), Pigou (1920), and Robertson (1952). In the context of the measurement of inequality of income distribution, and in that of judging alternative distributions of income, it has been used by Dalton (1920), Lange (1938),

[20] The logical and intuitive bases of alternative frameworks of interpersonal comparisons of individual welfare are discussed in some detail in Sen (1970), Chapter 7, 7*, 9 and 9*.

Lerner (1944), Aigner and Heins (1967), and Tinbergen (1970), among others.[21]

The trouble with this approach is that maximizing the sum of individual utilities is supremely unconcerned with the interpersonal distribution of that sum. This should make it a particularly unsuitable approach to use for measuring or judging inequality. Interestingly enough, however, not only has utilitarianism been fairly widely used for distributional judgements, it has—somewhat amazingly—even developed the reputation of being an egalitarian criterion. This seems to have come about through a peculiar dialectical process whereby such adherents of utilitarianism as Marshall and Pigou were attacked by Robbins and others for their supposedly egalitarian use of the utilitarian framework. This gave utilitarianism a ready-made reputation for being equality-conscious.

The whole thing arises from a very special coincidence under some extremely simple assumptions. The maximization of the sum of individual utilities through the distribution of a given total of income between different persons requires equating the marginal utilities from income of different persons, and if the special assumption is made that everyone has the same utility function, then equating marginal utilities amounts to equating total utilities as well. Marshall and others noted this particular aspect of utilitarianism, though they were in no particular hurry to draw any radical distributive policy prescription out of this. But when the attack on utilitarianism came, this particular aspect of it was singled out for an especially stern rebuke.

While this dialectical process gave utilitarianism its ill-deserved egalitarian reputation, the true character of that approach can be seen quite easily by considering a case where one person A derives exactly twice as much utility as person B from any given level of income, say, because B has some

[21] There is the question as to whether it is correct to identify individual 'welfare levels' with individual 'utilities' as the classical utilitarians saw these concepts. On this see Little (1950), Robertson (1952), and Sen (1970). In this work, I shall treat the two as identical following the traditional practice in economics.

handicap, e.g., being a cripple. In the framework of interpersonal comparisons outlined earlier, this simply means that the person making the judgement regards A's position as being twice as good as B's position for any given level of income. In this case the rule of maximizing the sum-total of utility of the two would require that person A be given a higher income than B. It may be noted that, even if income were equally divided, under the assumptions made A would have received more utility than B; and instead of reducing this inequality, the utilitarian rule of distribution compounds it by giving more income to A, who is already better off.

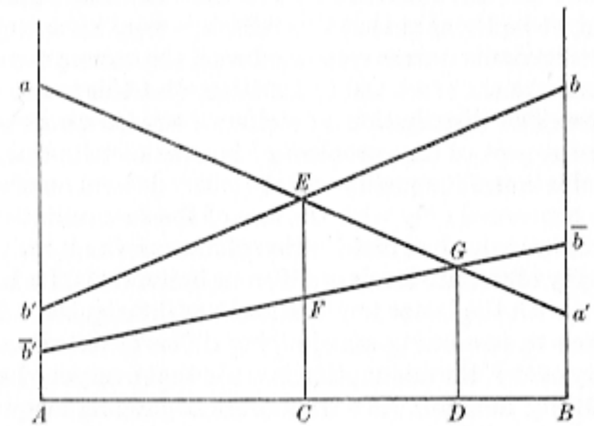


DIAGRAM 1.1

Diagram 1.1 illustrates the problem. The total amount of income to be divided between the two is AB. The share of A is measured in the direction AB and that of B in the direction BA and any point such as C or D reflects a particular division of total income between the two. The marginal utility of A is measured by aa' and that of B by bb', and as drawn they are exact mirror-images of each other. The maximum total of utility is secured by dividing income equally as given by point C with $AC = BC$. So far so good. Assume now that B's marginal utility schedule is exactly half that of A, so that his marginal utility is no longer given by bb' but by $\bar{b}\bar{b}'$. If the income distribution is left unchanged, A's total utility will be $AaEC$ and B's only $B\bar{b}FC$, and B will be much worse off. To compensate this an egalitarian criterion will now shift income from A to B. Would utilitarianism recommend this? It would recommend precisely the opposite, viz., a transfer of income from poor B to rich A! The new optimal point will be D with A enjoying a total utility of $AaGD$ and B merely $B\bar{b}GD$.

It seems fairly clear that fundamentally utilitarianism is very far from an egalitarian approach. It is, therefore, odd that virtually all attempts at measuring inequality from a welfare point of view, or exercises in deriving optimal distributional rules, have concentrated on the utilitarian approach.

It might be thought that this criticism would not apply at all if utilitarianism were combined with the assumption that everyone has the same utility function. But this is not quite the case. The distribution of welfare between persons is a relevant aspect of any problem of income distribution, and our evaluation of inequality will obviously depend on whether we are concerned only with the loss of the *sum* of individual utilities through a bad distribution of income, or also with the inequality of welfare levels of different individuals. Its lack of concern with the latter tends to make utilitarianism a blunt approach to measuring and judging different extents of inequality even if the assumption is made that everyone has the same utility function. As a framework of judging inequality, utilitarianism is indeed a non-starter, despite the spell that this approach seems to have cast on this branch of normative economics.

The Weak Equity Axiom

To bring in egalitarian considerations into the form of the social welfare judgements, we might propose various alternative axioms, of which the following is an interesting case.

The Weak Equity Axiom: Let person i have a lower level of welfare than person j for each level of individual income. Then in distributing a given total of income among n individuals including i and j, the optimal solution must give i a higher level of income than j.

This Axiom, which we shall call WEA, puts a restriction on the class of group welfare functions that can be considered. Note that the requirement does not specify how much more is to be given to the deprived person but merely that he should receive more income as a compensation, possibly partial, and even a minute extra amount would satisfy WEA. In this sense the requirement is rather mild.

Three qualifications should be specified here. First, the normative appeal of WEA would very likely depend on the precise interpretation of interpersonal comparisons. The framework in terms of which WEA seems to me to make a great deal of sense is the one that is being used in this work, viz., considering the possibility of being in different persons' positions and then choosing among them. Thus interpreted WEA amounts to saying that if I feel that for any given level of income I would prefer to be in the position of person A (with his tastes and his other non-income characteristics) than in that of person B, then I should recommend that B should get a higher income level than A.

Second, the more equity-conscious one is and the less concerned with the 'aggregate', the more should WEA appeal. It might be argued that if a unit of income gives much more marginal utility to A than to B despite A being in general better off than B, perhaps one should give the additional unit of income to A rather than to B. This type of 'marginalist' comparison is in the spirit of utilitarianism, whereas the philosophy behind WEA lies in a completely different direction. Part of the difference is purely normative, but there are technical problems of measurability and interpersonal comparability that have a bearing on this and which will be discussed in Chapter 2.

Third, it is possible to give person B so much more income that, despite having a lower welfare function, he may end up being much better off than person A. Such possibilities are not ruled out by WEA. It just indicates a *direction* of adjustment, but if the adjustment is quantitatively excessive the inequality may well finish up in the opposite direction. Other conditions have to be introduced to rule out such an occurrence. WEA is a pretty mild force towards equity, and it is at best a necessary but not a sufficient condition for achieving that objective.

It is clear from our earlier discussion that utilitarianism will violate WEA in many cases. Indeed the example portrayed in Diagram 1.1 shows this quite convincingly. To keep track of the more significant analytical results we elevate this piece of rustic wisdom into a theorem.

Theorem 1.2

There exist social choice situations such that the utilitarian rule of choice would violate the Weak Equity Axiom.

The proof is straightforward; look again at Diagram 1.1.[22]

WEA and concavity

The utilitarian rule is to maximize simply the sum of individual utilities:

$$W = \sum_{i=1}^{n} U_i(x) \tag{1.1}$$

To bring in a built-in bias towards equality, the functional relation between social welfare W and individual utilities may be assumed to be strictly concave. That is if U^1 and U^2 are two n-tuples of individual utilities, then for any t with $0 < t < 1$, we may require that:

$$[t\, W(U^1) + (1-t)\, W(U^2)] < W(tU^1 + (1-t)U^2) \tag{1.2}$$

This would imply that any 'averaging' of utilities, thereby reducing disparity, would tend to raise social welfare, which does of course push us in the egalitarian direction.

It is interesting to enquire into the relation between the Weak Equity Axiom and strict concavity, since both have egalitarian aspects. It should be clear, however, that the two conditions are in fact independent of each other. WEA is a condition of *optimal* choice in a restricted class of choice situations, and there is no real hope of being able to get the fulfilment of strict concavity (or even weaker conditions like

[22] Utilitarianism can satisfy WEA only if the ranking of total utilities is the opposite of that of marginal utilities at equal levels of income.

concavity or quasi-concavity) everywhere in the W-function on the basis of these restricted choice results.[23]

What about the converse? This does not follow either. WEA would require that the inequality-increasing result of the utilitarian case in the situation portrayed in Diagram 1.1 would have to be completely knocked out and instead the inequality-decreasing result brought in. And a W-function that is strictly concave in a mild way and is very close to the linear W of the utilitarian case would not be able to do this.

If not convinced by this reasoning, consider the following example. Take two utility functions identical except for a proportional displacement:

$$U_2(y) = m\, U_1(y), \text{ for all income level } y, \text{ with } m < 1 \tag{1.3}$$

Assuming that $U_1(y)$ is strictly positive for all positive values of y, person 2 is worse off than person 1 for all y. The group welfare function W is of the following form:

$$W = \frac{1}{\alpha}[(U_1)^\alpha + (U_2)^\alpha] \tag{1.4}$$

For strict concavity we need $\alpha < 1$. If the problem is to maximize W subject to:

$$y_1 + y_2 \leq Y, \tag{1.5}$$

the optimal distribution would have the property:

$$[U(y_1)]^{\alpha-1}\, U'(y_1) - [U(y_2)]^{\alpha-1}\, U'(y_2)\, m^\alpha = 0, \tag{1.6}$$

putting $U(y) = U_1(y)$. This implies:

$$U'(y_1)/U'(y_2) = m^\alpha\, [U(y_1)/U(y_2)]^{1-\alpha} \tag{1.7}$$

Note that if $\alpha > 0$, then $m^\alpha < 1$, and if $\alpha < 1$, then $m^\alpha > 1$. Since U increases and U' decreases with income y, it is clear that this condition will fulfil $y_1 < y_2$ if and only if $\alpha < 0$.[24]

[23] Note also that in some cases (see footnote 22) it is possible for WEA to be satisfied by utilitarianism, which must always violate strict concavity.

[24] In this case social welfare W is bounded from above. There is an analogy here with Ramsey's (1928) social welfare picture with a level of 'bliss'.

Since strict concavity only requires $\alpha < 1$, and WEA, in this case, would be fulfilled only if $\alpha < 0$, obviously WEA does not follow from strict concavity.[25]

Equity and welfare economics

WEA and the requirement of strict concavity have the common property of being in conflict with utilitarianism for essentially egalitarian reasons. But they do differ from each other in the way they bring in egalitarian values. Strict concavity does it by putting in a preference for the averaging process everywhere in the social welfare valuation, but the preference could be quite mild. In contrast WEA demands a sharper preference for equality in optimal choices but only for a specific class of situations. The two conditions are similar in spirit, but one is weak and widespread and the other is somewhat stronger but more confined in its scope.

It is worth mentioning in this context that in rejecting utilitarianism we have made use here of very mild conditions. Much stronger egalitarian criteria have been proposed before, e.g., Rawls's (1958), (1971) 'maximin' rule, whereby the social objective is to maximize the welfare level of the worst-off individual. In a 2-person world, WEA is a much weaker requirement than that of Rawls. If one person has a uniformly lower welfare function than another, and if the first person can be made better off by transferring income from the second, then the Rawls criterion would require that the person with the lower welfare function should have *that much* more income which would make his actual level of utility equal with that of the other.[26] In contrast, WEA merely requires that the

[25] If $\alpha > 0$, then $y_1 > y_2$, and if $\alpha = 0$, which is the borderline logarithmic case, $y_1 = y_2$.

[26] This requirement cannot, of course, be satisfied if the person with the lower welfare function remains worse off than the other despite having all the income, in which case he *should* have all the income according to the Rawls criterion. This is not a very interesting case for obvious reasons. In fact, it can be argued that the pure model of distribution involving costless transfers is, in general, not very relevant for practical policy making and how egalitarian the Rawls criterion would, in fact, be would depend upon the dependence of the total income on its distribution. Problems of incentives and related matters are discussed in Chapter 4.

unfortunate person should get a bit more—how much more is not specified. The fact that utilitarianism cannot even clear such a small hurdle seems to make it inappropriate as a theory for evaluating inequality.[27]

To conclude, we do not seem to get very much help in studying inequality from the main schools of welfare economics—old and new. The literature on Pareto optimality (including the famous 'basic theorem' of 'new' welfare economics) avoids distributional judgements altogether. The standard approach of 'social welfare functions' because of its concentration on individual orderings only (without any use of interpersonal comparisons of levels and intensities) fails to provide a framework for distributional discussions. This is brought out rather dramatically by Theorem 1.1. Finally, utilitarianism, the dominant faith of 'old' welfare economics, is much too hooked on the welfare *sum* to be concerned with the problem of distribution, and it is, in fact, capable of producing strongly anti-egalitarian results. As an approach to the measurement and evaluation of inequality it cannot take us very far. For the problem of inequality evaluation, the royal roads of welfare economics do look a trifle bleak.

[27] Note that the conflict with utilitarianism would have arisen even if WEA had demanded only that the person with the uniformly lower welfare function should get *no less* income than the other.

2

Measures of Inequality

In this chapter I shall discuss a number of measures of inequality that have been proposed in the literature. As mentioned in the last chapter, these measures fall into two classes, viz., positive measures which make no explicit use of any concept of social welfare, and normative measures which are based on an explicit formulation of social welfare and the loss incurred from unequal distribution. While I did argue that the line between these two types of measures is not a firm one, it is clear that there is a distinction, and it may be useful to discuss the two types of measures in turn. I shall begin with positive measures.

The range

Consider distributions of income over n persons, $i = 1, \ldots n$, and let y_i be the income of person i. Let the average level of income be μ, so that:

$$\sum_{i=1}^{n} y_i = n\mu \tag{2.1}$$

The relative share of income going to person i is x_i. That is:

$$y_i = n\mu x_i \tag{2.2}$$

Perhaps the simplest measure is based on comparing the extreme values of the distribution, i.e., the highest and the lowest income levels. The range can be defined as the gap between these two levels as a ratio of mean income. Thus defined the range E is given by:

$$E = (\text{Max}_i\ y_i - \text{Min}_i\ y_i)/\mu \tag{2.3}$$

If income is divided absolutely equally, then clearly $E = 0$. At the other extreme if one person receives all the income, then $E = n$. And E lies in general between 0 and n.

The difficulty with the range as a measure is obvious enough. It ignores the distribution in between the extremes.

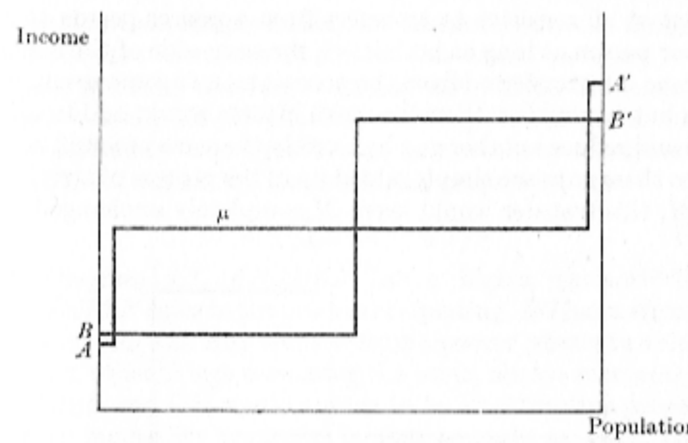


Diagram 2.1

The distribution AA' has a wider range E than BB', but most people under AA' enjoy the mean income μ with only a few aberrations. On the other hand BB' involves a division of the population into two distinct classes of the rich and the poor. By concentrating on the extreme values only, the range misses important features of the contrast.

The relative mean deviation

One way of looking at the entire distribution and not merely at the extreme values is to compare the income level of each with the mean income, to sum the absolute values of all the differences, and then to look at that sum as a proportion of total income. This yields the so-called relative mean deviation M:

$$M = \sum_{i=1}^{n} |\mu - y_i|/n\mu \tag{2.4}$$

With perfect equality $M = 0$, and with all income going to one person only, $M = 2(n-1)/n$. But unlike E, M takes note of the entire distribution. For example, in Diagram 2.1 the value of M is much higher for BB' than for AA', which corresponds well to our intuitive notion of inequality.

The main trouble with the relative mean deviation is that it is not at all sensitive to transfers from a poorer person to a richer person as long as both lie on the same side of the mean income. £1 transferred from the poorest man to someone more rich but having less than the mean income would add to one gap and reduce another gap by exactly the same amount, and since these gaps are simply added up in the process of arriving at M, this transfer would leave M completely unchanged.

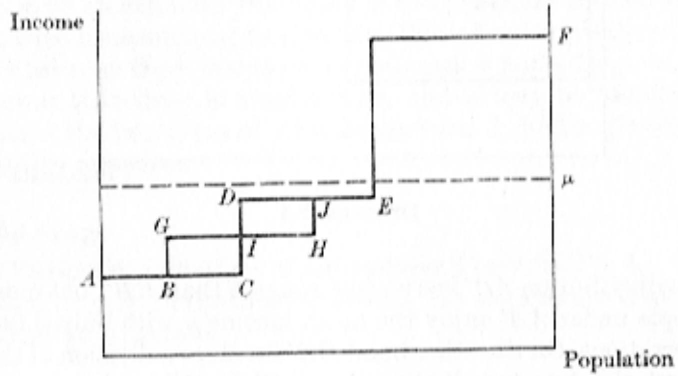


Diagram 2.2

In Diagram 2.2 the distribution $ABCDEF$ is transformed into $ABGHJEF$ by transferring income to some of the poorest from a richer class. But the value of M remains unchanged since the diminution of the gap by $BGIC$ is exactly compensated by the increase of the gap by $DIHJ$, since—as drawn—BC and DJ are equal and so are BG and JH. As a measure M seems to take no notice of income transfers whatsoever unless they cross the dividing line of μ on the way. It is, therefore, rather arbitrary—a bit like some criminal laws in the United States which come into operation only if some state boundary

is crossed in the process of the crime. As a measure M fails to catch the commonly accepted ideas on inequality, which would tend to regard $ABCDEF$ as more unequal than $ABGHJEF$.

The variance and the coefficient of variation

Rather than simply adding the absolute values of the gaps, if we square them and then add, this would have the result of accentuating differences further away from the mean, so that a transfer like the one shown in Diagram 2.2 would reduce the inequality measure. Variance, the common statistical measure of variation, does have this property.

$$V = \sum_{i=1}^{n} (\mu - y_i)^2/n \qquad (2.5)$$

In Diagram 2.2, $ABCDEF$ has a higher variance than $ABGHJEF$ since in the process of squaring, BG has a stronger impact than JH. Any transfer from a poorer person to a richer person, other things remaining the same, always increases the variance, and this would appear to be an attractive property for an inequality measure. In fact as early as 1920, Hugh Dalton had argued that any measure of inequality must have this minimal property[1] and since in this Dalton was following a lead of Pigou,[2] whom he quoted in this context, we shall call this the Pigou–Dalton condition.

However, the variance depends on the mean income level, and one distribution may show much greater *relative* variation than another and still end up having a lower variance if the mean income level around which the variations take place is smaller than with the other distribution. A measure that does not have this deficiency and concentrates on relative variation is the coefficient of variation, which is simply the square root of the variance divided by the mean income level:

$$C = V^{\frac{1}{2}}/\mu \qquad (2.6)$$

While the coefficient of variation captures the property of being sensitive to income transfers for all income levels and,

[1] Dalton (1920), p. 351.
[2] Pigou (1912), p. 24.

unlike the variance, is independent of the mean income level, the procedure of squaring the differences is a very particular one. And the question may be asked: Why choose this particular formula? It is easily checked that C does have the characteristic of attaching equal weights to transfers of income at different income levels, i.e., the impact of a small transfer from a person with income y to one with income $(y - d)$ is the same, irrespective of the value of y.[3] Is this neutrality a desirable property? It is possible to argue that the impact should be greater if the transfer takes place at a lower income level, and a transfer from a person with an income level of £1,000 to one with £900 should be greater than a similar transfer from a man with £1,000,100 to one with £1,000,000. However, by now we are dealing with areas in which our intuitive ideas of inequality are relatively vague and checking the measures in terms of some commonly accepted notions of inequality is no longer altogether easy. But the question remains: Why use the squaring procedure rather than some other operation which would also make the inequality measure sensitive to transfers from the rich to the poor (in line with the Pigou–Dalton condition)?

There is another methodological issue. Is it best to measure the difference of each income level from the mean only, or should the comparison be carried out between every pair of incomes? The latter will capture everyone's income difference from everyone else, and not merely from the mean, which might not be anybody's income whatsoever.

The standard deviation of logarithms

If one wishes to attach greater importance to income transfers at the lower end, a reasonable way of going about it is to take some transformation of incomes that staggers the income levels, and of course the logarithm recommends itself. One other advantage of the logarithm, in contrast with taking the variance or the standard deviation of *actual* values, is that it eliminates the arbitrariness of the units and therefore of absolute levels, since a change of units, which takes the form of a multiplication of the absolute values, comes out in the logarithmic form as an addition of a constant, and therefore goes out in the wash when pairwise differences are being taken. It is, therefore, no wonder that the standard deviation of the logarithm has frequently cropped up as a suggested measure of inequality. As used in the standard statistical literature, the deviation is taken from the geometric mean rather than from the arithmetic mean, but in the income distribution literature using the arithmetic mean seems more common (see Atkinson 1970, Stark 1972).

$$H = [\textstyle\sum_{i=1}^{n}(\log \mu - \log y_i)^2/n]^{\frac{1}{2}} \tag{2.7}$$

The fact that a logarithmic transformation staggers the income levels tends to soften the blow in reflecting inequality since it reduces the deviation, but on the other hand it has the property—as noted before—of highlighting differences at the lower end of the scale. But since income levels, as they get higher and higher, suffer increasingly severe contraction, this makes $-H$ as a measure of welfare not concave at all at high income levels. If one wants social welfare to be a concave function of individual incomes, then H as a measure of inequality can cause problems, despite attractive features in other respects.

Furthermore, H does depend on the arbitrary squaring formula—albeit after a logarithmic transformation—and it shares with V and C the limitation of taking differences only from the mean.

The Gini coefficient and the relative mean difference

A measure that has been very widely used to represent the extent of inequality is the Gini coefficient attributed to Gini (1912) and much analysed by Ricci (1916) and later by Dalton (1920), Yntema (1938), Atkinson (1970), Newbery (1970), Sheshinski (1972), and others. One way of viewing it is in terms of the Lorenz curve due—not surprisingly—to Lorenz (1905), whereby the percentages of the population arranged

[3] Atkinson (1970), p. 255.

from the poorest to the richest are represented on the horizontal axis and the percentages of income enjoyed by the bottom x% of the population is shown on the vertical axis.

Obviously 0% of the population enjoys 0% of the income and 100% of the population enjoy all the income. So a Lorenz curve runs from one corner of the unit square to the diametrically opposite corner. If everyone has the same income the Lorenz curve will be simply the diagonal, but in the absence of perfect equality the bottom income groups will enjoy a proportionately lower share of income. It is obvious, therefore, that any Lorenz curve must lie below the diagonal (except the

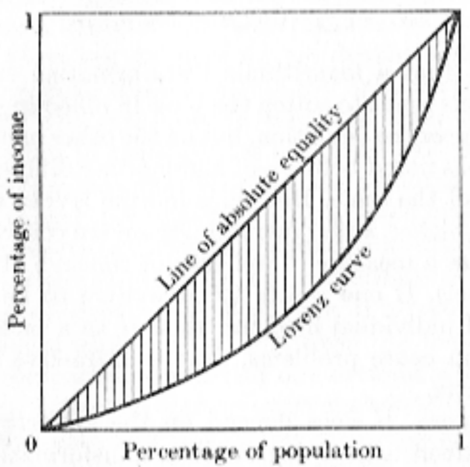


DIAGRAM 2.3

one of complete equality which would be the diagonal), and its slope will increasingly rise—at any rate not fall—as we move to richer and richer sections of the population.

The Gini coefficient is the ratio of the difference between the line of absolute equality (the diagonal) and the Lorenz curve —represented in Diagram 2.3 as the shaded area—to the triangular region underneath the diagonal. There are various ways of defining the Gini coefficient, and a bit of manipulation —tedious as it is—reveals that it is exactly one-half of the

relative mean difference, which is defined as the arithmetic average of the absolute values of differences between all pairs of incomes.

$$G = (1/2n^2\mu) \sum_{i=1}^{n} \sum_{j=1}^{n} |y_i - y_j| \quad (2.8.1)$$

$$= 1 - (1/n^2\mu) \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{Min}(y_i, y_j) \quad (2.8.2)$$

$$= 1 + (1/n) - (2/n^2\mu) [y_1 + 2y_2 + \ldots + ny_n] \quad (2.8.3)$$

$$\text{for } y_1 \geq y_2 \geq \ldots \geq y_n.$$

In taking differences over all pairs of incomes, the Gini coefficient or the absolute mean difference avoids the total concentration on differences vis-à-vis the mean which C, V, or H has. In avoiding the arbitrary squaring procedure of C, V, or H, it may seem to be a more direct approach as well, without sacrificing the quality of being sensitive to transfers from the rich to the poor at every level. Undoubtedly one appeal of the Gini coefficient, or of the relative mean difference, lies in the fact that it is a very direct measure of income difference, taking note of differences between *every* pair of incomes.

Welfare interpretations of the alternative measures

In comparing these measures it is obvious that the range E and the relative mean deviation M are, more or less, non-starters.[4] The real competition would be between such measures as the coefficient of variation C, the standard deviation of logarithms H and the Gini coefficient G. In comparing the relative usefulness of these measures it is necessary to examine the precise properties.

First, as far as the Pigou–Dalton condition is concerned, both the coefficient of variation and the Gini coefficient, pass the test, i.e., a transfer from a richer man to a poorer person always reduces the value of both C and G. The same is,

[4] Measures of 'skewness' of income distributions have also been used as measures of inequality. But this is essentially a confusion of 'equality' with 'symmetry'. An un-skewed symmetric distribution need not be an equal one. Cf. Stark (1972), pp. 139–140.

however, not true of the standard deviation of logarithms H, and it is possible for H to rise even when there are rich-to-poor transfers. Although this can happen only at very high levels of income, the fact remains that H can violate the Pigou–Dalton condition.[5]

Second, as far as relative sensitivity is concerned, we have already noted that the coefficient of variation C is equally sensitive at all levels, whereas the standard deviation of logarithms H is more sensitive for transfers in the lower income brackets. As noted earlier, if the welfare impact of a tiny transfer from a man with £1,000 to one with £900 is thought to be more important than that from a man with £100,100 to one with £100,000, the coefficient of variation faces some problems. The standard deviation of logarithms shows precisely the required type of sensitivity, but it becomes so insensitive to transfers *among* the rich, that it may end up by violating even the Pigou–Dalton condition as well. It would have been nice if the sensitivity direction could be preserved (unlike with C) without violating the Pigou–Dalton condition (unlike with H). Would the Gini coefficient meet this gap?

The answer is: No, it will not. For the sensitivity of the Gini coefficient depends not on the size of the income levels but on the number of people in between them. As is clear from the expression (2.8.3), the Gini coefficient implies a welfare function which is just a weighted sum of different people's income levels with the weights being determined by the rank-order position of the person in the ranking by income level. Thus the rate of substitution between the person with the i-th highest income and the one with the j-th highest income is simply j/i. For example £3 to the second richest person is given the same weight as £2 to the third richest man. So the actual weights would depend upon precisely how the population is distributed over income sizes. If A has an income of £2,000 and B of £1,900, and if A is the 1,000th richest man while B is the 1,100th richest man, then £1·00 to B is taken to be equivalent to £1·10 to A. But if some other people turn up inside the income

[5] See Dasgupta, Sen and Starrett (1972), and also Chapter 3 below.

gap, e.g., if an additional 100 people get incomes between £1,900 and £2,000, then the Gini coefficient would attach the same weight to £1·00 income to B as to £1·20 to A. The income levels of A and B have remained the same, but the relative weighting between them is now completely altered because some other people have shown up inside the income range defined by A's and B's income levels.[6]

The Gini coefficient can be interpreted in a number of different ways. The visual picture given by Diagram 2.3 is itself quite expressive. The expression (2.8.3), on which we have been commenting, shows that the implicit welfare function underlying the Gini coefficient is a rank-order-weighted sum of different persons' income shares. On the other hand, expression (2.8.2) throws a somewhat different light on the Gini coefficient. Suppose the welfare level of any pair of individuals is equated to the welfare level of the worse-off person of the two.[7] Then if the total welfare of the group is identified with the sum of the welfare levels of all pairs, we get the welfare function underlying the Gini coefficient.

Finally, expression (2.8.1) suggests yet another interpretation. In any pair-wise comparison the man with the lower income can be thought to be suffering from some depression on finding his income to be lower. Let this depression be proportional to the difference in income. The sum total of all such depressions in all possible pair-wise comparisons takes us to the Gini coefficient.

One characteristic of the Gini coefficient is that it does not imply a strictly concave group welfare function.[8] This is obvious from (2.8.3), since G is a linear function of income

[6] There is an obvious analogy here with the violation of Arrow's (1951) 'independence of irrelevant alternatives', which we have used in Chapter 1. The similarity is not a pure coincidence. A rank-order-based system typically does make choices sensitive to 'irrelevant' alternatives, and this is as true of the 'rank-order method' of voting, which violates Arrow's condition, as of the Gini coefficient, which has the property described in the text.

[7] There is an analogy here with Rawls's (1971) 'maximin' criterion of justice but applied pair-wise.

[8] Note that the welfare function must be thought to be $-G$, since a higher value of G shows *greater* inequality, which corresponds to *less* welfare. It is $-G$ that is concave but not strictly concave.

levels. This property has come under attack recently,[9] but it is not at all clear how serious an objection it really is. The implied group welfare function may not be strictly concave, but it is concave all right, and furthermore any transfer from the poor to the rich *or* vice versa is strictly recorded in the Gini measure in the appropriate direction.[10] The fact that the standard deviation of logarithms does not even satisfy this condition of response may appear to be, in some sense, much more clearly objectionable.[11]

Theil's entropy measure

An interesting measure of inequality, proposed by Theil (1967), derives from the notion of entropy in information theory, and it is, in terms of motivation, rather different from the class of measures we have been looking at. When x is the probability that a certain event will occur, the information content $h(x)$ of noticing that the event has in fact occurred must be a decreasing function of x—the more unlikely an event, the more interesting it is to know that that thing has really happened.

[9] See Atkinson (1970), Newbery (1970), Dasgupta, Sen and Starrett (1970). Newbery (1970) shows that the Gini coefficient cannot order distributions in the same way as any *additive* group welfare function given strictly concave and differentiable individual utility functions. Sheshinski (1972) questions additivity and gives an example of a non-additive group welfare function that would reflect the Gini ranking. Dasgupta, Sen and Starrett (1972) and Rothschild and Stiglitz (1972) demonstrate that the Gini ranking cannot be reflected by any group welfare function (additive or not) if it is strictly quasi-concave on individual incomes. (Sheshinski's example is not only non-additive, but it is also non-strictly-quasi-concave on incomes.)

[10] The Gini coefficient is strictly S-concave though not strictly concave, or strictly quasi-concave. On this see Dasgupta, Sen and Starrett (1972), and Chapter 3 below.

[11] Measures like the variance and the coefficient of variation are strictly concave throughout. But the use of the mean-variance analysis in an additive framework would imply a very specific class of individual utility functions, viz., the quadratic class:

$$U(y) = k_1 + K_2 y + K_3 y^2, \text{ with } K_3 < 0 \tag{2.9}$$

The coefficient of variation is, however, mean-independent, though it too would fall in a relatively narrow category in terms of implicit welfare functions.

One formula that satisfies this property—among others—is the logarithm of the reciprocal of x.

$$h(x) = \log \frac{1}{x} \tag{2.10}$$

When there are n possible events $1, \ldots, n$, we take the respective probabilities $x_1, \ldots, x_n$, such that $x_i \geq 0$ and $\Sigma_{i=1}^n x_i = 1$. The entropy or the expected information content of the situation can be viewed as the sum of the information content of each event weighted by the respective probabilities.

$$H(x) = \Sigma_{i=1}^n x_i h(x_i)$$
$$= \Sigma_{i=1}^n x_i \log\left(\frac{1}{x_i}\right) \tag{2.11}$$

It is clear that the closer the n probabilities x_i are to $(1/n)$, the greater is the entropy. While in thermodynamics entropy is taken to measure disorder,[12] if x_i is interpreted as the share of income going to person i, $H(x)$ looks like a measure of equality. When each x_i equals $(1/n)$, $H(x)$ attains its maximum value of $\log n$. If we subtract the entropy $H(x)$ of an income distribution from its maximum value of $\log n$, we get an index of inequality. This is Theil's measure.

$$T = \log n - H(x)$$
$$= \Sigma_{i=1}^n x_i \log nx_i. \tag{2.12}$$

Given the association of doom with entropy in the context of thermodynamics, it may take a little time to get used to entropy as a good thing ('How grand, entropy is on the increase!'), but it is clear that Theil's ingenious measure has much to be commended. A shift from a richer to a poorer person lowers T, i.e., it satisfies the Pigou–Dalton condition, and it can be aggregated in a simple manner over groups.[13] But

[12] The second law of thermodynamics warns that there is an inherent tendency for disorder to increase.

[13] Theil (1967), pp. 94–6.

the fact remains that it is an arbitrary formula, and the average of the logarithms of the reciprocals of income shares weighted by income shares is not a measure that is exactly overflowing with intuitive sense.[14] It is, however, interesting that the concept of entropy used in the natural sciences can provide a measure of inequality that is not immediately dismissable, however arbitrary it may be.

Different mean incomes

It is important to note that all these measures with the exception of the case of the variance have the property of being invariant if everyone's income is raised in the same proportion. This is true of the relative mean deviation, the coefficient of variation, the standard deviation of the logarithms, the relative mean difference, the Gini coefficient, and Theil's entropy measure. Is this a property we want? Can it be asserted that our judgement of the extent of inequality will not vary according to whether the people involved are generally poor or generally rich? Some have taken the view that our concern with inequality increases as a society gets prosperous since the society can 'afford' to be inequality-conscious. Others have asserted that the poorer an economy, the more 'disastrous' the consequences of inequality, so that inequality measures should be sharper for low average income.

This is a fairly complex question and is bedevilled by a mixture of positive and normative considerations. The view that for poorer economies inequality measures must be themselves sharper can be contrasted with the view that greater *importance* must be attached to any given inequality measure if the economy is poorer. The former incorporates the value in question into the measure of inequality itself, while the latter brings it in through the evaluation of the relative importance of a given measure at different levels of average income. In a fundamental sense it does not really matter whether these

[14] If for some reason the individual welfare functions are proportional to $x_i \log(1/x_i)$, then Theil's measure will be particularly attractive within the utilitarian framework, but I can't think of a good reason why individual welfare functions should take such a form.

values are brought in through the measure itself or through the evaluation of the measure, but it is important to be clear about precisely what one is doing.

Normative

Dalton's measure

It is time now to move from the positive measures to normative ones. In a classic contribution, Dalton (1920) argued that any measure of economic inequality must be concerned with economic welfare to be of relevance. The particular measure that he chose followed directly from the utilitarian framework, and he based it on a comparison between actual levels of aggregate utility and the level of total utility that would obtain if income were equally divided. Since he took a strictly concave utility function, i.e., with diminishing marginal utility of income, and the same function for all, the maximization of aggregate welfare required an equal division. Dalton took the ratio of actual social welfare to the maximal social welfare as his measure of equality, taking the utility levels to be all positive.[15]

$$D = [\textstyle\sum_{i=1}^{n} U(y_i)]/nU(\mu). \qquad (2.13)$$

Atkinson (1970) has pointed out that this measure suffers from the difficulty that it is not invariant with respect to positive linear transformations of the utility function; cardinal utility implies that any positive linear transformation would do just as well and Dalton's measure takes arbitrary values depending on which particular transformation is chosen. I must confess that I am not entirely persuaded that this argument is a very strong one, since the ordering of Dalton's measure would not be affected by taking positive linear transformations, and what is really significant with these measures is the ordering property. However, it is possible to redefine the measure in such a way that the actual numbers used in measuring would be invariant with respect to permitted transformations of the welfare numbers, and this is what Atkinson does in his own approach.

[15] See Wedgwood (1939) for an application of Dalton's measure. See Bentzel (1970) for a critical evaluation of this approach.

Atkinson's measure

Atkinson defines what he calls 'the equally distributed equivalent income' of a given distribution of a total income, and this is defined as that level of *per capita* income which if enjoyed by everybody would make total welfare exactly equal to the total welfare generated by the actual income distribution.[16] Putting y_e as 'the equally distributed equivalent income', we see that:

$$y_e = y|[nU(y) = \textstyle\sum_{i=1}^{n} U(y_i)] \qquad (2.14)$$

The sum of the actual welfare levels of all equals the welfare sum that would emerge if everyone had y_e income. Since each $U(y)$ is taken to be concave, i.e., with non-increasing marginal utility, y_e cannot be larger than the mean income μ. Further, it can be shown that the more equal the distribution the closer will y_e be to μ. Atkinson's measure of inequality is:

$$A = 1 - (y_e/\mu). \qquad (2.15)$$

Obviously if income is equally distributed then y_e is equal to μ, and the value of Atkinson's measure will be 0. For any distribution the value of A must lie between 0 and 1.

There are some difficulties with Atkinson's measure which relate to the problems that we discussed in the last chapter. To begin with, a relatively simple problem. Atkinson requires that the function $U(y)$ be concave but not necessarily strictly concave, i.e., $U' > 0$ and $U'' \leq 0$.[17] Consider two distributions between two persons with a given total amount of income, say, (0, 10) and (5, 5). If we choose a $U(y)$ function such that it is proportional to y, both will have precisely the same Atkinson measure of inequality. However, it would seem to be rather absurd to describe the two as being equally unequal.

We are confronting two distinct problems here. First, being based exclusively on a normative formulation, the measure of

[16] For an earlier use of the approach of 'equally distributed equivalent income', see Champernowne (1952), one of whose measures was 'the proportion of total income that is absorbed in compensating for the loss of aggregate satisfaction due to inequality' (p. 610).

[17] Atkinson (1970), pp. 245–6.

inequality has ceased to have the descriptive content that is associated with it in normal usage, and the idea of inequality has become totally dependent on the form of the welfare function. Since under the assumptions both distributions produce the same level of social welfare, they appear to have the same measure of inequality. But, of course, in the sense in which the word inequality is used in normal communication, it has a straightforward descriptive content as well. And it would be odd to describe (0, 10) and (5, 5) as having the same degree of inequality. The second problem concerns the use of the utilitarian framework whereby the values of U of each person are simply added to arrive at the aggregate social welfare. If, instead of that, social welfare were taken to be a strictly concave function of individual utilities—in the line suggested in the last chapter—then these two distributions would not have had the same measure of inequality and indeed (0, 10) would have been more unequal than (5, 5).

Of course, Atkinson himself is careful not to call his $U(y)$ a utility function. Perhaps we can even think of it as some kind of a strictly concave transform of individual utilities, i.e., the component of social welfare corresponding to person i, being itself a strictly concave function of individual utilities. What one would have to do then is to require strict concavity of the $U(y)$ function.

Axioms for additive separability

However, while this takes care of the problem of strong concavity, it is still fairly restrictive to think of social welfare as a sum of individual welfare components. There are in fact two separate issues in utilitarianism, viz., the question of simply adding the individual utilities and the question of additive separability; the former implies the latter but not the other way around.[18] If we take each U as a strictly concave function of individual utilities we are avoiding the simple additive formula of utilitarianism, but we are still sticking to the notion

[18] The group welfare function given by equation (1.4) in Chapter 1 is additively separable, but not utilitarian except for the special case of $\alpha = 1$.

of additive separability. Individual components of social welfare continue to be judged without reference to the welfare components of others, and the social welfare components corresponding to different persons are eventually added up to arrive at an aggregate value of social welfare.

There are various ways of axiomatizing additive separability in the context of income distributional judgements.[19] An interesting version of this is presented by Hamada (1972) in an illuminating paper in which he proceeds in terms of an analogy with behaviour under risk. Though Hamada's model is rather complex, it is worth examining carefully because the requirements of additive separability are brought out sharply by his axiom set.

Consider incomes (taken for convenience to be integers) ranging from 1 to m, which is the maximum possible income. Let r_i be the percentage of population receiving income i, for $i = 1, \ldots, m$. So any income distribution can be represented by $(r_1, \ldots, r_n)$, which will be called vector r. Hamada calls this 'the income distribution vector', but this may cause confusion in our context since r is not a vector of income but of percentages of population. I shall call r a Hamada-vector. Obviously the sum of the components equals 1, i.e., $\Sigma_i r_i = 1$. Consider two Hamada-vectors r and s. Split each into two vectors r^1 and r^2, and s^1 and s^2, respectively, i.e., $r^1 + r^2 = r$, and $s^1 + s^2 = s$. Assume that $\Sigma r^1_i = \Sigma s^1_i$, and therefore $\Sigma_i r^2_i = \Sigma_i s^2_i$. Multiply each of these vectors r^1 and s^1 by a suitable number such that the resultant vectors $\hat{r}^1$ and $\hat{s}^1$ are Hamada-vectors, i.e., $\Sigma_i r^1_i = \Sigma_i s^1_i = 1$. Do the same to r^2 and s^2 to get Hamada-vectors $\hat{r}^2$ and $\hat{s}^2$. The crucial axiom that Hamada uses (his Assumption 2) requires that if we regard $\hat{r}^1$ to be at least as good as $\hat{s}^1$ and $\hat{r}^2$ at least as good as $\hat{s}^2$, then we must regard r to be at least as good as s. Moreover, if we strictly prefer $\hat{r}^1$ to $\hat{s}^1$ and regard $\hat{r}^2$ to be at least as good as $\hat{s}^2$, then we must strictly prefer r to s.

Is this a reasonable axiom for income distribution? Consider two Hamada-vectors (0, 100, 0, 0) and (50, 0, 0, 50). The former, which we shall call $\hat{r}^1$, has absolute equality at income 2 for each, while $\hat{s}^1$ is a two-class society with the 'haves' getting income 4 each while the 'have-nots' make do with only income 1 each. Despite the higher total income of the latter, it is possible that someone impressed by the vision of total equality in $\hat{r}^1$ will swear by Babeuf that it is superior to $\hat{s}^1$. Next, this man is given a choice between (39, 22, 0, 39) and (40, 20, 0, 40), which we shall call respectively r and s. All that vision of equality is now gone, and in this rather mundane choice it is possible that the same man may not really be able to say that he prefers r to s. While r is a bit more of an equal distribution, s does have a bit more of total income, and in going from s to r one person each is taken out from income groups 1 and 4 respectively to be put into income group 2, but that still leaves 39 others at the two poles. Our hero may not, thus, be too impressed with r and may not prefer r to s. But then he has had it with Hamada. We can split r into (0, 2, 0, 0), and (39, 20, 0, 39), which we call r^1 and r^2 respectively, and s into (1, 0, 0, 1) and (39, 20, 0, 39), which—as our hero would have by now guessed—we are going to call s^1 and s^2 respectively. Normalized into Hamada-vectors r^1 and s^1 reveal themselves to be $\hat{r}^1$ and $\hat{s}^1$, and we know that our hero prefers $\hat{r}^1$ to $\hat{s}^1$. He must be indifferent between $\hat{r}^2$ and $\hat{s}^2$, since they are the same. So he must prefer r to s by Hamada's axiom, but no, he doesn't. Our little hero gets into difficulty with additive separability because he is taking an *interdependent* view of income distribution.

In general, if one feels that the social valuation of the welfare of individuals should depend crucially on the levels of welfare (or incomes) of others,[20] this property of the independence of each person's welfare component from the position of others has to be sacrificed. And this requires the use of a less narrow class of social welfare functions. Hamada's axiom system precipitates this independence property in a clear form, but these axioms are both sufficient *and* necessary for additive separability, and the difficulty is in fact quite pervasive.

[19] On the general question of additive separability, see Debreu (1960), Gorman (1968), Fishburn (1970), and Hammond (1972). See also the controversies between Strotz (1958), (1961), and Fisher and Rothenberg (1961), (1962), and between Harsanyi (1955) and Diamond (1967), on related issues.

[20] Cf. Runciman (1966) on 'relative deprivation'.

An alternative measure

Consider social welfare W to be an increasing function of individual income levels:

$$W = W(y_1, \ldots, y_n).$$

A more general normative measure of inequality is the following. Define y_f (the generalized equally distributed equivalent income) as that level of *per capita* income which if shared by all would produce the same W as the value of W generated by the actual distribution of income.

$$y_f = y|[W(y, \ldots, y) = W(y_1, \ldots, y_n)] \qquad (2.16)$$

Assuming W to be symmetric and quasi-concave, y_f would be less than or equal to μ for every distribution of income. In this more general form, W need not even be a function of individual utilities, i.e., it need not even be 'individualistic'.

The measure of inequality that we can use with this more general approach will now be given by:

$$N = 1 - (y_f/\mu) \qquad (2.17)$$

It is quite clear that A and N given by (2.15) and (2.17) will be completely equivalent if the welfare function to be used is of the utilitarian form:

$$W = \sum_{i=1}^{n} U(y_i)$$

Positive and normative measures

If we choose a general formula like N for measuring inequality of income distribution we have considerable freedom about specifying the social welfare function. We can then choose whatever we think are the appropriate assumptions about the valuation of the welfare implications of inequality. Any use of N will of course require a specification of the W function. While N avoids the problem of additive separability, it is a totally normative measure—a characteristic that it shares with Dalton's and Atkinson's measures. But, as argued in the last chapter, inequality measures do have positive elements which are difficult to disassociate from the welfare picture.

In some ways the positive measures of inequality discussed earlier can also be viewed as normative measures with specific assumptions about social welfare evaluation. For example, Theil's entropy measure is almost strictly in the form of a utilitarian social welfare function which makes the individual welfare components equal to $x_i \log(1/x_i)$, where x_i is the share of income going to person i. This is a rather peculiar welfare function, and the other measures could be justified in normative terms also with rather special representations of social welfare. I do not intend to go through each of these measures one by one to see what they translate into; some of the welfare aspects of measures like V, C, H and G were discussed earlier. There is no great analytical difficulty in putting all the measures within the same framework and then differentiating between them in terms of their normative assumptions. But this is not a fair thing to do as far as the positive measures are concerned, since the motivation underlying these measures is quite different.

Assumptions of measurability and comparability

The rationale for using a general measure like N is its independence from the narrow framework of additively separable individualistic group welfare functions. The utilitarian welfare function corresponds to the most common example of this narrow framework and, given utilitarianism, A and N would coincide, and D would also generate the same ordering. The relevance of utilitarianism to the problem of normative evaluation of income distributions is, therefore, of significance. While evaluating the merits of utilitarianism in Chapter 1, I made the assumption of cardinal measurability and full interpersonal comparability of individual welfares. It is worth investigating whether a defence of utilitarianism can rest on some other specific measurability and comparability assumptions.

The lack of equity considerations in the utilitarian framework (combined with the case for strict concavity of social welfare on individual welfare levels) was spelt out in Chapter 1. The Weak Equity Axiom was chosen as an illustration of an equity-conscious requirement. There are others one can suggest. But all these requirements—and indeed the very idea of equity—would seem to need comparisons of levels of welfare in addition to comparisons of differences. If one assumed a framework whereby gains and losses of welfare of different individuals could be compared but absolute levels could not be, it would be possible to use the utilitarian approach without being able to apply WEA or other equity considerations of that type. This is because the utilitarian formula declares x to be socially preferred to y if and only if the welfare differences for all the individuals between x and y summed together turn out to be positive, and in defining differences the question of the 'origin' of the utility function is irrelevant. If, for example, a certain constant is added to one person's utility level but not to that of another, it will not affect the utilitarian rule of choice at all. But it can affect the ordering of different persons' welfare levels, thereby rendering the WEA and other equity conditions unusable.

Let W_U stand for any 'utilitarian' group welfare function and W_E any group welfare function satisfying WEA or some similar equity condition involving comparisons of relative

Measurability \ Comparability	Non-comparability	Only units comparable	Only levels comparable	Units and levels both comparable
Ordinal	W_U and W_E both unusable	X	W_E usable, not W_U	X
Cardinal	W_U and W_E both unusable	W_U usable, not W_E	W_E usable, not W_U	W_U and W_E both usable

W_U = any 'utilitarian' group welfare function
W_E = any 'equity based' group welfare function (e.g., satisfying WEA)

TABLE 2.1

wellbeing of different people. In Table 2.1 the consequences of different measurability and comparability assumptions are presented.

It is clear that, should we assume ordinality of individual welfare, there would be no possibility whatsoever of using utilitarianism, while equity conditions like WEA could still be used if ordinal interpersonal comparability were permitted. On the other hand, if we take cardinality of individual welfare and assume that welfare differences are comparable, but not levels, then utilitarianism can be used, but equity conditions like WEA cannot be.

The real conflict comes when both levels and differences are comparable and the assumption of cardinality with full comparability puts utilitarianism in potential conflict with WEA and other equity-conscious rules, since both are usable and they can be mutually inconsistent. The arguments used in the last chapter concentrated on this case and I shall not repeat them. It seems to me clear that a concern for equity must militate against the use of utilitarianism.

Inequality judgements and comparability

In defending utilitarianism, therefore, one's best bet would be in making the rather peculiar assumption that utility differences are comparable but levels are not.[21] In this context it is important to bear in mind the meaning of interpersonal comparisons of utility which was discussed in the last chapter. As we argued there, to judge that $U_i(x) > U_j(y)$ is the same thing as preferring to be individual i in state x rather than individual j in state y. These *as if* comparisons are an essential part of thinking about the normative aspects of the problem of income distribution. It is, of course, certainly the case that these comparisons are subjective, but they are, I would submit, a particularly relevant way of understanding problems of equity. And in so far as putting oneself in the position of another includes the exercise of having his tastes and mental make-up as well, the exercise may indeed be quite complex. But in understanding the interpersonal aspects of social choice

[21] Cf. Harsanyi (1955), Diamond (1967), and Sen (1970), pp. 143–6

involving human wellbeing, one is inevitably put in the position of having to make these comparisons, though very often this is done implicitly. The advantage of making the comparisons explicitly is to crystalize one's values on the subject, and this is a significant part of any normative recommendation in a situation of conflict between the interests of different individuals or groups.

Formally, if we think of the ordering $\tilde{R}$ over elements like (x, i), i.e., being person i in state x, we can think of interpersonal welfare comparisons as simply numerical reflections of such an ordering $\tilde{R}$. As discussed in Chapter 1:

$$U_i(x) \geq U_i(y) \text{ if and only if}$$

$$(x, i)\ \tilde{R}\ (y, j). \qquad (2.18)$$

If the problem is viewed in this way it should be clear that comparisons of levels are *prior* to comparisons of differences. Indeed, the utility functions will have to be first defined in terms of the ranking $\tilde{R}$, which immediately introduces comparability of *levels*, and only if, on top of that, some additional characteristics (like 'independence') are satisfied, would differences be also comparable in an interpersonally usable cardinal scale. It would thus seem that there is rather little scope for defending utilitarianism by making the assumption of comparability of differences of utility with levels being non-comparable.

3

Inequality as a Quasi-Ordering

It can be argued that part of the difficulty in using the measures of inequality presented in the last chapter arises from the fact that they are all 'complete' measures in the sense that every pair of distributions can be compared under each of these measures. If we take any two distributions x and y—each being a vector of incomes—then according to any of these criteria either x will be more unequal than y, or vice versa, or both will be equally unequal. The possibility of non-comparability is not at all entertained. In fact, to each distribution x there is attached—under any of these measures—a real number $I(x)$ which is supposed to represent the degree of inequality of x. While various ways of arriving at $I(x)$ have been presented (e.g., using specific 'positive' measures such as the coefficient of variation, or the standard deviation of logarithms, or the Gini coefficient, or using particular measures of welfare loss such as $(1-D)$, A, or N, after specifying the relevant welfare functions), each way leads to the conversion of the set of distributions x, y, z, etc., into a set of corresponding inequality numbers $I(x)$, $I(y)$, $I(z)$, and so on. And since any two numbers are comparable, i.e., either $I(x) > I(y)$, or $I(x) < I(y)$, or $I(x) = I(y)$, there is never any gap in the picture of comparative inequality.

It is, however, possible to argue that this approach is inherently defective since inequality as a notion does not have any innate property of 'completeness'. In a trivial sense it is, of course, the case that one can define 'inequality' precisely as one likes, and as long as one is explicit and consistent one may think that one is above criticism. But the force of the expression 'inequality', and indeed our interest in the concept, derive from the meaning that is associated with the term, and we are not really free to define it purely arbitrarily. And—as it happens—the concept of inequality has different facets which may point in different directions, and sometimes a total ranking can not be expected to emerge. However, each of the standard measures does yield a complete chain, and arbitrariness is bound to slip into the process of stretching a partial ranking into a complete ordering. It is arguable that each of these measures leads to some rather absurd results precisely because each of them aims at giving a complete-ordering representation to a concept that is essentially one of partial ranking.

Lorenz partial ordering and Atkinson's results

One measure of inequality that does not aim at 'completeness' is the relation of the Lorenz curve of one distribution being strictly inside that of another. In Diagram 3.1 the Lorenz curve x lies wholly inside curve z, and so does curve y

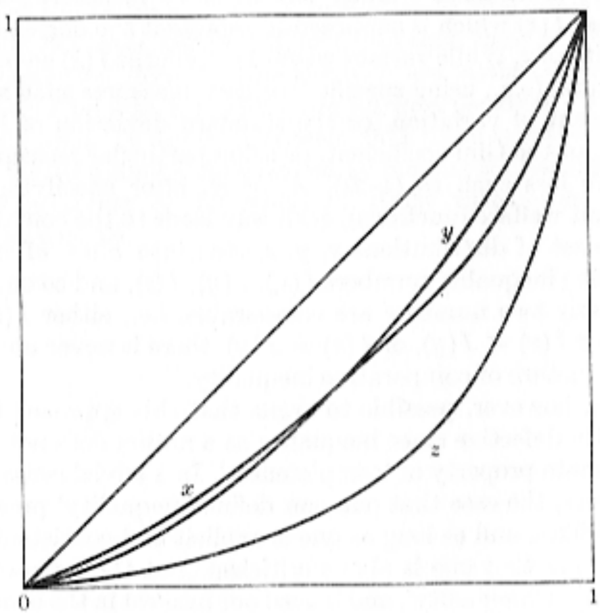


DIAGRAM 3.1

as well. But the curves x and y intersect, so that neither can be said to be more unequal than the other in terms of the Lorenz-curve ranking. Treating L as the relation of being strictly inside,[1] we can say xLz and yLz, but neither xLy, nor yLx.

Are there reasons to believe that the Lorenz-curve ranking catches the essence of the concept of inequality, including its partial nature? Up to a point the answer probably is 'yes', and I propose now to discuss the nature of the Lorenz ranking L in some detail. Later on, however, it will be shown that it does miss some essential features of the concept of inequality.

A remarkable theorem on the Lorenz ranking was proved by Atkinson (1970), using the normative approach. Suppose that social welfare is the sum of individual U functions which are themselves strictly concave functions of income y_i, i.e., have strictly diminishing marginal utility.

$$W(y) = \sum_{i=1}^{n} U(y_i) \tag{3.1}$$

Let the Lorenz curve of distribution x lie strictly inside that of y, i.e., xLy. The total income is the same for both distributions. Then even without knowing which precise U function is used, we can say that $W(x)$ is greater than $W(y)$, where $W(x)$ and $W(y)$ are the social welfare levels from x and y respectively. Furthermore, the converse is also true, i.e., if we can say that $W(x) > W(y)$ irrespective of which individual U function is chosen (as long as it is strictly concave), then xLy. Thus xLy implies $W(x) > W(y)$ irrespective of the precise concave utility function chosen, and if for all strictly concave utility functions $W(x) > W(y)$, then xLy.

Non-additive formulation

The great attraction of Atkinson's result is that it permits us to rank the inequality levels of distributions in terms of the social welfare levels even without knowing the precise utility function to be chosen. However, since social welfare is taken to be of the utilitarian additive kind, the result may be thought

[1] This is taken to mean that xLy if the Lorenz curve of x lies nowhere outside that of y and at some place (at least) it lies strictly inside the latter,

to be somewhat limited. This is not merely because simply adding individual utilities is a very dubious procedure for arriving at social welfare, but also because even the relatively less demanding assumption of additive separability is quite restrictive, as discussed in the preceding chapters.

What will happen if the class of social welfare functions is extended to include non-additive ones as well? We can define social welfare W simply as a symmetric and concave function of individual welfare levels while the individual welfare functions are strictly concave.

$$W = G(U(y_1), \ldots, U(y_n)). \tag{3.2}$$

It stands to reason that Atkinson's results can be extended to this case as well. After all, if xLy leads to $W(x) > W(y)$, even when we are simply adding individual utilities, that tendency must be strengthened if we make W a concave function of individual welfare levels (preserving symmetry), with diminishing relative importance of individual welfare as we consider richer and richer men. This could only *reinforce* the tendency towards $W(x) > W(y)$. Having a concave social welfare function defined on individual welfare levels gives a further egalitarian bias, and this adds to the egalitarian tendencies arising from concave individual welfare functions. It would thus appear that Atkinson's results must be generalizable in terms of a wider class of social welfare functions, of which his additive function will be a special case.

Non-individualistic welfare functions

There is another respect in which it may be useful to view the problem in more general terms than Atkinson has done. It is possible to view social welfare in non-individualistic terms, i.e., not relating social welfare to individual utilities as such. For example, social welfare may be defined directly on the distribution of incomes without going through the intermediary of individual utilities. The distinction has been discussed earlier in Chapter 1. Thus defined we can think of social welfare being given by a function of the following type:

$$W = F(y_1, \ldots, y_n). \tag{3.3}$$

It is clear that, given the individual utility function U, the individualistic form of the function as given by G in the earlier formulation is really a special case of this. That is, even when using a function like F, we can go, if we like, through the intermediary of individual utilities, but we are not obliged to do so.

There are at least two different reasons for preferring a more general function of the kind of F. First, the planner, or the social critic, or the political leader, or whoever is making the distributional judgement, may under certain circumstances feel inclined to bypass individual preferences. There is, perhaps, a 'paternalistic' element in this, but such a thing is frequently present in policy discussions. The argument may relate to considerations of individual 'irrationality', 'shortsightedness', and similar matters, and how seriously we entertain these possibilities remains an open question. Second, sometimes the person making the distributional judgement might simply not have detailed information on individual utility functions. Under these circumstances, even though one might prefer to go via individual utilities, it might not be practically possible to do so. It may be then necessary to deal with the function of the type of F dispensing with the unworkable intermediary.

How convincing these arguments are, I do not wish to debate. The question is not crucial for our purpose, since we lose nothing in our exercises by operating on functions like F rather than G in view of the fact that G is a special case of F. In what follows, therefore, I shall stick to the more general form.

Weakening of concavity

A third respect in which the Atkinson picture can be extended concerns the restrictions to be imposed on the concavity of the welfare functions. It may be recalled that, since social welfare is defined in the Atkinson framework as a sum of strictly concave individual U functions (the same function for all), translated into the F-form the social welfare function F will be strictly concave. However, for incorporating the egalitarian bias for distributional judgements, it is sufficient to consider strict quasi-concavity.[2]

The distinction between concavity and quasi-concavity is a technical one, but is worth commenting on. A concave welfare function F requires that the weighted average of social welfare levels from two income distributions x and y must be less than or equal to the social welfare of the weighted average of the two distributions, using the same weights.

$$tF(x) + (1-t)F(y) \leq F(tx + (1-t)y), \quad \text{for any } t,\ 0 < t < 1. \tag{3.4}$$

On the other hand quasi-concavity requires that the *minimum* of the two social welfare levels from x and y respectively should be less than or equal to the social welfare of the weighted average of the two distributions.

$$\text{Min}\,[F(x), F(y)] \leq F(tx + (1-t)y), \quad \text{for any } t,\ 0 < t < 1. \tag{3.5}$$

For *strict* quasi-concavity the weak inequality $\leq$ is to be replaced by $<$, so that the social welfare from the weighted average must be strictly larger than the minimum of the two welfare levels from x and y respectively.

Essentially strict quasi-concavity simply requires that the social indifference curves (for more than two persons, social indifference surfaces) must be themselves concave outwards, i.e., be shaped like curved dishes. That this follows immediately from the definition is clear from Diagram 3.2, where the two axes y_1 and y_2 represent the income levels of the two persons. Since x and y lie on the same indifference curve they must have the same level of social welfare, and therefore the minimum social welfare of the two must be the social welfare from either. z is a weighted average of the two distributions x and y, and strict quasi-concavity requires that social welfare from

[2] In fact, a bit of further weakening is possible, viz., requiring only S-concavity (on this, see Berge, 1963), used in the extension of the Atkinson theorem presented in Dasgupta, Sen and Starrett (1972).

z be strictly larger than social welfare from x or y. That is, the social indifference curve through z must be higher than the one through x and y. This is guaranteed if and only if the social indifference curves are shaped concave outwards. Strict quasi-concavity means that, as we increase the income level of one

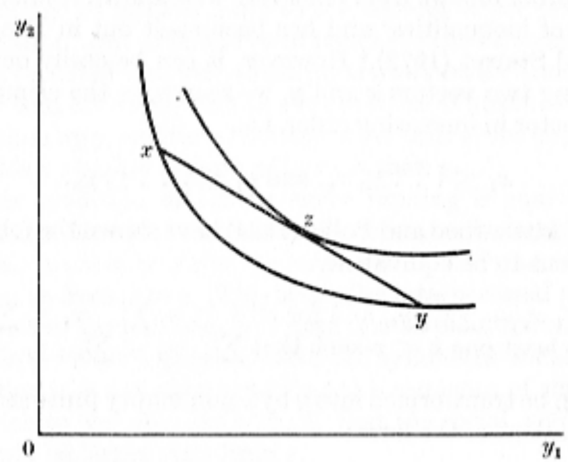


DIAGRAM 3.2

given the income levels of the others, less and less relative importance is attached to the income level of the person whose income is going up. This is a strictly egalitarian feature, which is all we need for building equality-consciousness into our social welfare function.

A general result

Considering, therefore, the social welfare function F defined over individual incomes, implying neither the necessity to go through the intermediary of individual utilities, nor the use of the utilitarian additive framework, nor even the necessity of strict concavity, let F be simply any function that is symmetric and strictly quasi-concave. The following theorem is true.

Theorem 3.1

Taking F to be symmetric and strictly quasi-concave, if for two different distributions x and y with the same total of income, yLx, then $F(y) > F(x)$, and if *not* yLx, then for some F, $F(y) \leq F(x)$.

The proof follows from relatively well-known results in the theory of inequalities[3] and has been spelt out in Dasgupta, Sen and Starret (1972).[4] However, it can be easily outlined.

Taking two vectors x and y, we rearrange the elements of each vector in increasing order, i.e.,

$$x_1 \leq \ldots \leq x_n, \text{ and } y_1 \leq \ldots \leq y_n.$$

Hardy, Littlewood and Polya (1934) have shown the following conditions to be equivalent.

(1) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$, and for all $k \leq n$, $\sum_{i=1}^{k} x_i \leq \sum_{i=1}^{k} y_i$, with at least one $k < n$ such that $\sum_{i=1}^{k} x_i < \sum_{i=1}^{k} y_i$. (3.6)

(2) x can be transformed into y by a non-empty finite sequence of operations of the form:

$$x_i^{\alpha+1} = x_i^{\alpha} + e^{\alpha} \leq x_j^{\alpha},$$
$$x_j^{\alpha+1} = x_j^{\alpha} - e^{\alpha} \geq x_i^{\alpha},$$

for $i < j$ and $e^{\alpha} > 0$, with $x_k^{\alpha+1} = x_k^{\alpha}$ if $k \neq i, j$ (3.7)

(3) For any strictly concave real-valued function U,

$$\sum_{i=1}^{n} U(x_i) < \sum_{i=1}^{n} U(y_i). \quad (3.8)$$

(4) While y is not x, nor a permutation of x, there is a bistochastic matrix Q, such that:

$$y = Qx \quad (3.9)$$

[3] See Hardy, Littlewood and Polya (1934). Through an interesting linguistic coincidence their book is called *Inequalities*, which refers of course to $<$ and $\geq$, and not to the slums and Marie Antoinette.

[4] In fact in that paper only strict S-concavity is assumed of which a special case is strict quasi-concavity, for a symmetric function. See also Kolm (1969), and Rothschild and Stiglitz (1972).

These standard results in the theory of inequalities are exceedingly handy for Theorem 3.1 as well as for understanding the general properties of Lorenz partial orderings. It can be readily recognized that condition (1) is simply the statement that the Lorenz curve of y lies strictly inside the Lorenz curve of x. Since the Lorenz curve is computed by taking the percentage of income going to the bottom m per cent of the population, and since the total income is the same in the two cases, the set of inequalities simply shows that for some bottom m per cent of the population a lower share of income is yielded by x than by y, and for *all* bottom m per cent of the population, x yields no higher a share of income than y.

This condition of Lorenz-curve ranking is equivalent to condition (2), which can be readily seen as a finite sequence of transformations transferring income from the rich to the poor, taking us from x to y. (This is so *after* interpersonal permutations since the i-th man in x need not be the same as the i-th man in y.) With a quasi-concave and symmetric social welfare function it is not surprising that this sequence of shifts from the rich to the poor must imply that the social welfare from y would be larger than from x.

Condition (3) takes us back to the Atkinson framework and shows that if y has a higher Lorenz curve than x, then any additive social welfare function with the same strictly concave U function for all individuals must yield a higher total social welfare in y than in x. Furthermore, since condition (3) is not only implied by condition (1) but also implies condition (1), it also follows that *not* (1) implies *not* (3). Therefore, Theorem 3.1 must obviously be true, viz., that if not yLx, then for some strictly concave U function, $\Sigma_i U(y_i) \leq \Sigma_i U(x_i)$. This Atkinson case being a special case of a strictly quasi-concave and symmetric F function, it is clear that if not yLx, then for some admissible F, $F(y) \leq F(x)$. So only the first part of the theorem remains to be proved.

Condition (4) is the only one with some technical content. A bistochastic matrix is a square matrix, all of the entries of which are non-negative and each of the rows and columns of which adds up to one. Multiplying a vector x by a bistochastic matrix Q converts it into another vector y, which also has the same sum of its elements taken together. A special case of a bistochastic matrix is a permutation matrix which simply re-orders the elements of a vector, i.e., permutes them. It is well known that any bistochastic matrix of order n is some convex combination of the set of permutation matrices of order n.[5] With P^s being any permutation matrix we can obtain Q from the set of such permutation matrices thus:

$$Q = \sum_s a_s P^s, \sum_s a_s = 1 \text{ and each } a_s \geq 0. \qquad (3.10)$$

Therefore, y lies inside the convex hull of the permutations of x, i.e., in the convex hull of the set $(P^s x)$ for all s. But y is not an extreme point of this convex hull. So y can be obtained as a convex combination of the set of permutations of x, which themselves are socially indifferent to each other, by virtue of symmetry. It follows immediately that for any strictly quasi-concave F satisfying symmetry:[6]

$$F(y) > F(x). \qquad (3.11)$$

Intuitive explanation

The last result indicates that if y has a higher Lorenz curve than x, then it must yield a higher social welfare than x for all symmetric, strictly quasi-concave group welfare functions. The last part of the proof, which is the only technical bit, may, however, be understood intuitively quite easily by considering the three person case. Diagram 3.3 presents such a picture with the three axes representing the income of the three individuals. Viewing the picture as a three-dimensional one (some exercise of imagination is certainly called for here), the shaded triangle ABC can be seen to be a portion of a plane

[5] See Berge (1963), p. 182, on the 'Theorem of Birkhoff and von Neumann'.

[6] Strict S-concavity is defined in the following way: F is strictly S-concave if and only if for all bistochastic matrices Q, $F(Qx) > F(x)$, if Qx is not x, nor a permutation of it. (3.11) follows directly from (3.9) given strict S-concavity and strict quasi-concavity is not really necessary. Cf. Dasgupta, Sen and Starrett (1972). Note, however, that Qx can be a permutation of x without Q being a permutation matrix.

caught between the three axes and lying slanted in this three-dimensional diagram with the characteristic that for any point on it the sum of its coordinates equals unity (i.e., it is the so-called 'unit simplex').

Consider, now, this triangle ABC on its own (Diagram 3.4). The distribution x will be a point on this triangle, assuming

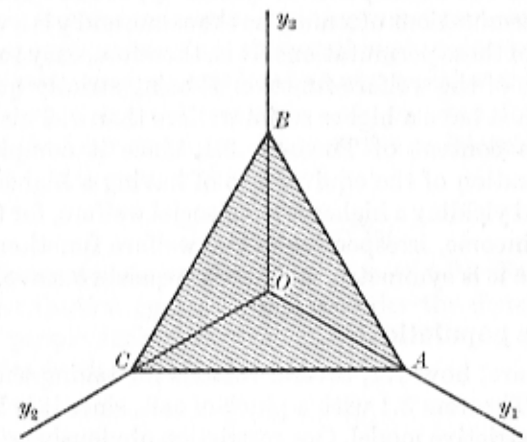


Diagram 3.3

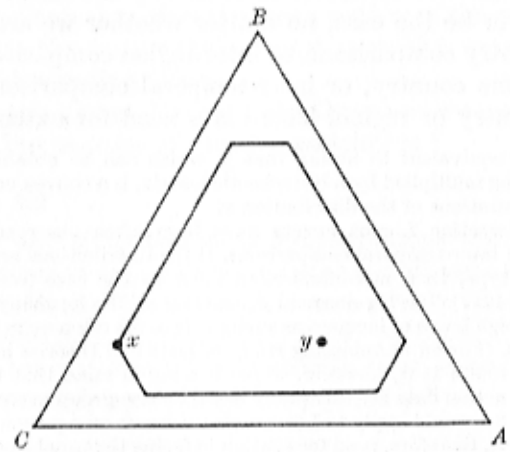


Diagram 3.4

that the total income to be distributed is unity, which is just a matter of choosing units. It is clear that in the three-dimensional case there are six distributions of income which are exact interpersonal permutations of the distribution x, and they are represented as the six corner points of the hexagon drawn through x. The point y will lie inside this hexagon.[7] Thanks to the assumption of symmetry, social welfare from all the permutations of x must be the same, and y is a weighted average of these permutations. It is, therefore, easy to see that by virtue of the welfare function F being strictly quasi-concave y must have a higher social welfare than x. This really is the main content of Theorem 3.1, since it completes the demonstration of the equivalence of having a higher Lorenz curve and yielding a higher level of social welfare, for the same total of income, irrespective of the welfare function chosen, as long as it is symmetric and strictly quasi-concave.[8]

Variable population

There are, however, several reasons for taking the significance of Theorem 3.1 with a pinch of salt, since it is based on a very restrictive model. One restriction obviously arises from the fact that we are assuming that the number of people involved in the two distributions is exactly the same. This will hardly ever be the case, no matter whether we are making inter-country comparisons, or inter-region comparisons within the same country, or inter-temporal comparisons in the same country or region. There is a need for extending the

[7] This is equivalent to noting that y, which can be obtained from x through being multiplied by a bistochastic matrix, is a convex combination of the permutations of the distribution x.

[8] Non-intesecting Lorenz curves have been often observed in inter-country and inter-temporal comparisons. If the distributions are all of the log-normal type, then non-intersection must be the case (see Aitchison and Brown, 1957). The log-normal form gives good fits for many countries, though for high levels of incomes as such the best fits often seem to take the Pareto-form. (For an illuminating study of facts and theories in this field, see Lydall, 1966.) It is, however, worth bearing in mind that the Lorenz curves from actual data are invariably based on size-group averages whereas Theorem 3.1 would apply to Lorenz curves drawn on a person-by-person basis. There is, therefore, need for caution in facing the usual Lorenz curves armed only with Theorem 3.1.

results presented in Theorem 3.1 to the case of variable population. This, as it happens, is not a very difficult thing to do *provided* we accept a relatively unobjectionable assumption.

Consider two countries with exactly identical populations and income distributions. Obviously they both must have the same level of social welfare and the same *per capita* welfare. If we now consider the two countries together rather than separately it stands to reason that they must continue to have the same *per capita* welfare, since nothing has changed except that the two are now considered together rather than separately. Generalizing this reasoning, we can put forward an axiom in the following form, denoting the social welfare function for a community with n people as:

$$W = F^n(y_1, \ldots, y_n). \tag{3.12}$$

The Symmetry Axiom for Population (SAP): For any income distribution $(y_1, \ldots, y_n)$, consider the distribution x over nr people such that $x_i = x_{2i} = \ldots = x_{ri} = y_i$, for $1 \leq i \leq n$, with r any integer. Then

$$F^{nr}(x) = rF^n(y). \tag{3.13}$$

What this axiom demands is simply that if r countries with the same population and identical income distributions are considered together, then the mean welfare of the whole must be equal to the mean welfare of each part. This would seem to be an undemanding axiom.

Given this axiom, however, the Lorenz-curve result can be extended to the case of variable population.[9]

Theorem 3.2

Let y^1 and y^2 be two income distributions with the same mean income over population sizes n^1 and n^2 respectively and let y^1Ly^2. Each F^n is symmetric and strictly quasi-concave and satisfies SAP. Then $(F^{n^1}/n^1) > (F^{n^2}/n^2)$. And if *not* y^1Ly^2, then for some symmetric and strictly quasi-concave welfare functions satisfying SAP, $(F^{n^1}/n^1) \leq (F^{n^2}/n^2)$.

[9] Strict quasi-concavity can be replaced by strict S-concavity; see Dasgupta, Sen and Starrett (1972).

The proof of this theorem is not difficult to devise. Consider a country with a population size of n^1n^2. Let it have an income distribution exactly like y^1 with each member in that distribution being replicated n^2 times. Consider a second hypothetical country with n^1n^2 population size with an identical distribution to that of y^2, each member of the latter being replicated n^1 times. Obviously hypothetical country 1 and hypothetical country 2 both have exactly the same Lorenz curves as y^1 and y^2 respectively,[10] and if $y^1 L y^2$, then hypothetical country 1 also has a higher Lorenz curve than hypothetical country 2. Both the hypothetical countries have, of course, the same population and the same total income. Therefore, by Theorem 3.1 the social welfare of hypothetical country 1 would be larger than the social welfare of hypothetical country 2. Now, we know from the symmetry axiom that the mean welfare of hypothetical country 1 must be the same as the mean welfare of actual country 1 and the mean welfare of the hypothetical country 2 must equal the mean welfare of actual country 2. Thus country 1 must have a higher mean welfare than country 2, which proves the first part of the theorem. The second part of the theorem follows from a similar construction, again using Theorem 3.1.

Mean income variations

This extension takes care of the problem of variable population. Lorenz-curve rankings seem to make good sense in comparisons of mean welfare even when the population size is a variable. However, the problem of variable mean income still remains. It would, of course, be possible to tackle the problem in a similar way to that of variable population by making a corresponding axiom. However, while SAP is, I think, quite defendable, the corresponding symmetry axiom for income will not be, since welfare may not be homogeneous with respect to the size of income. It is obvious that any possibility of making distributional judgements independently of the size of

[10] There is, in fact, some ambiguity as to how we define Lorenz curves in the discrete case. For our purpose we can plot the Lorenz points for each discrete number of people and connect them by straight lines.

income will make sense only if the relative ordering of welfare levels of distributions were strictly neutral to the operation of multiplying everybody's income by a given number. We might not, however, wish to make this assumption, since our judgement about social welfare may not be scale-independent in this sense. Thus, the problem of extending the Lorenz partial ordering to cases of variable mean income is quite a serious one, and this—naturally enough—restricts severely the usefulness of this approach.

Since the art of practical economics often involves compromises it may be necessary sometimes to make Lorenz-curve comparisons for countries with different mean income, but it must be borne in mind that in reading welfare implications in such comparisons in the light of Theorems 3.1 and 3.2, one would have to bring in some symmetry axiom for income, which may not be particularly justifiable.

Non-compulsive judgements

This question relates generally to the case for viewing the usual income-distributional comparisons as 'non-compulsive judgements'. A non-compulsive judgement indicates the belief that there is a reason for acting in a certain way and that there is a *prima facie* case for that action. But it is not a compelling recommendation, and contrary reasons could be produced.[11] The fact that one distribution has a higher Lorenz curve than another can be taken to constitute a *prima facie* case that it is a better distribution from the welfare point of view. Of course, contrary arguments may exist, and variation of mean income may well be one such. But it seems reasonable to demand that someone rejecting the Lorenz results on these grounds must specify *how* he expects the differences in mean income to affect the distributional judgements from the welfare point of view. While the Lorenz ranking is not in itself compelling, the onus of demonstration may well be thought to lie on the person wishing to reject this ranking on other grounds.

[11] The distinction between 'compulsive' and 'non-compulsive' judgements is presented and analysed in Sen (1967).

Descriptive content

It should also be said in defence of the Lorenz judgements that the purely descriptive content of the Lorenz partial ordering is also not negligible. First of all, for the simple case of the same population size and the same total income, it may be recalled that the equivalence of conditions (1) and (2) outlined earlier means that y's having a higher Lorenz curve than x implies that one can transform x into y by shifting income from the rich to the poor. This is an unambiguous sense in which income distribution must be thought to be more equal in y than in x. Even without bringing in anything about welfare, a transfer from the rich to the poor must mean descriptively that the level of inequality has gone down, and thus a higher Lorenz curve must mean less inequality even in the purely descriptive sense.

The same picture is brought out also by Diagram 3.4 where y can be seen to be lying strictly *inside* the symmetric hexagon on which x lies. This is a purely descriptive feature, and while it has normative implications, the statement that y is less unequal than x can also be viewed as a factual one in terms of definitions corresponding closely to the normal usage of the term inequality.

The same feature survives the case of variable population as well, since everything can be done in terms of percentages of population, and once again the statement that y is less unequal than x would seem to be meaningful and acceptable. Stretching this picture to the case of variable mean income may not be too objectionable from the positive, as opposed to the normative, point of view. The triangle in Diagram 3.4 takes the total income as 1, and if two communities with different mean income are compared, we can still think of y being a less unequal distribution than x in *relative* terms. The fact that our welfare judgements may crucially depend on the size of income per head does not affect this picture since here we are concerned not with the normative features but with purely descriptive ones in relative terms. We could, of course, still say that while y represents a more equal relative distribution

than x from a descriptive point of view, the relative impact on welfare of the inequality level, low as it is, of y may still be larger than the consequence of the relatively higher level of inequality of x, because of differences of mean income. We would then be dissociating the measure of inequality from the judgement of its welfare implications.

As I have tried to outline earlier, income-distributional measures have these two distinct but interlinked features. Even in normal communication both the normative and positive aspects can be observed in the use of the concept of inequality. While the Lorenz relation catches both aspects, it seems to take a firmer grip of the descriptive aspect than it does of the normative, especially when the mean income level varies.

Inequality quasi-orderings

The Lorenz dominance relation yields a partial strict ordering. I had begun this chapter by arguing that there is a good general case for expressing our judgements on inequality in the form of quasi-orderings. What is the precise difference between a partial strict ordering and a quasi-ordering? The answer is: Not very much, but a quasi-ordering is 'reflexive', which a strict partial ordering is not, and the latter is 'asymmetric', which a quasi-ordering is not. Stripped of the technicalities this means roughly that a quasi-ordering is a relation like 'at least as unequal as', whereas a strict partial ordering is one like 'more unequal than'.[12] It is, of course, obvious that a slight extension will permit us to get a quasi-ordering out of the Lorenz partial ordering. What is perhaps more interesting is the fact that in the process the conditions imposed on the form of the group welfare function can also be relaxed further.

Being concerned with the weak inequality relation 'at least as unequal as', we look now for the welfare ranking 'being at least as great as' and not for the relation 'greater than', i.e.,

[12] Note that the former is 'reflexive' in the sense that any distribution x is, of course, 'at least as unequal' as itself, and not 'asymmetric' in the sense that x being 'at least as unequal' as y does not preclude the possibility that y may also be 'at least as unequal' as x, since the two distributions may be judged to be equally unequal. For 'more unequal than', the opposite holds.

we are interested in conditions that yield $F(y) \geq F(x)$ and not also $F(y) > F(x)$. This permits an immediate extension of the permitted class of group welfare functions from strictly quasi-concave ones to those simply quasi-concave (irrespective of whether they are strictly so or not).

Let yRx stand for either yLx, i.e., y being Lorenz-superior to x, or x and y being identical distributions. The latter we include to permit reflexivity of R, but we do not of course rule out the possibility that two distributions may be judged to be 'equally unequal' despite not being identical distributions.

Theorem 3.3

R is a quasi-ordering and furthermore yRx implies that:

for all symmetric and quasi-concave F: $F(y) \geq F(x)$.

It is obvious that R is reflexive and transitive. The rest of the proof follows from the weak set of equivalent inequalities corresponding to (1)–(4) used in proving Theorem 3.1 above,[13] and it need not be spelt out here.[14]

The judgement that R provides on inequality seems to be very broad-based indeed. From the purely descriptive point of view if yRx then y can be obtained from x either by permuting incomes between the individuals, or by a combination of that with a sequence of transfers of income from the richer to the poorer. Its normative justification is also based on very mild assumptions. Quasi-concavity will be satisfied if a transfer from the richer to the poorer *does not worsen* the welfare level (whether or not it improves it). This follows from the *absence* of positively anti-egalitarian values. And, of course, the other attractive features of Theorems 3.1 and 3.2 are retained, viz., there is no need to assume the additive framework of utilitarianism, or even additive separability, or for that matter an 'individualistic' group welfare function.

[13] In fact, the original versions used in Hardy, Littlewood and Polya (1934) were the weak ones.

[14] Note also that Theorem 3.3 can be extended to the case of S-concavity instead of requiring quasi-concavity. Further, *strict* S-concavity is no longer needed. S-concavity is defined as $F(Qx) \geq F(x)$ for all bistochastic matrices Q.

But there are gaps in this picture of normative solidarity. First, the idea that social welfare is a function of money incomes only is itself a very restrictive one. Consider the same distribution of money incomes, with a change in prices. Even if the price index, defined as some kind of a weighted average, remains the same, still the effective distribution of purchasing power can now be different, since price changes have a different impact on different people in view of the variation (i) in the tastes and (ii) in the money income levels of different persons. The former is obvious enough, but the latter is also easily seen. Even when everyone has the same tastes, if the price of food goes up a poorer person's welfare level goes down relatively more, since food represents a bigger part of his budget.

Second, variations of income may effectively limit the applicability of Theorems 3.1–3.3, for reasons that have already been spelt out. How should comparisons be made when the mean income level varies?

Finally, there is the question of the appropriateness of the symmetry property of the group welfare function and the assumption of equal needs. In Chapter 1 the relaxation of this assumption was shown to have crucial effects, but in the theorems covered in this chapter we have stuck to this axiom like a leech. I shall postpone further discussion of this last problem until the next chapter, when the concepts of deserts and needs will both be reviewed. But I intend to go into the first two problems now.

Price variations and inequality

The complexity caused by price variations may be taken up first. This is undoubtedly an important question, but we must try to avoid being mesmerized by its nihilistic pretensions. Indeed, the possibility of price variations, which is virtually always present in any comparison of two different situations, has frequently been used to rule out welfare judgements altogether and has thus been an effective means of terrorizing the egalitarian. But the analytical picture is, in fact, by no means so clear.

Consider an observed situation with the vector of money incomes y and the vector of prices p. Each pair (y_i, p) gives us the money income of person i as well as the prices at which he has spent that income. Evidently his welfare can be thought to be given by a utility function based on (y_i, p), in the absence of externalities, and more generally by a function of (y, p) even when person i is affected by the wellbeing and consumption of others. Social welfare can be defined over (y, p) either directly or through the intermediary of individual welfare levels.[15]

More generally, social welfare judgements can take the form of a ranking relation B defined over the set of pairs of (y, p). If (y, p^1) is regarded as at least as good as (x, p^2), we can write:

$$(y, p^1)\ B\ (x, p^2). \tag{3.14}$$

The relation B can be expected to be reflexive and transitive, i.e., to be a quasi-ordering. If B is also complete then the social welfare ranking would be an ordering. Given that, and with some additional assumptions,[16] we can define social welfare W as a real-valued function $E(y, p)$.

$$W = E(y, p). \tag{3.15}$$

If welfare judgements are not easy to make given the complexity of price comparisons, B may not be complete. It is one thing to say that we *can* make social welfare judgements based on y and p; it is quite another to say that we shall find it easy to formulate such judgements. Frequently they will be particularly difficult to make.[17] In some cases the contrast may be so

[15] Formally, each (y, p) is an element of the Cartesian product of the set of n-vectors of money incomes and the set of k-vectors of prices, when there are n people and k commodities.

[16] Cf. Chapter 1, footnote 7.

[17] For a penetrating analysis of the general question of distributional judgements in a many-commodity world, see Fisher (1956) and Kenen and Fisher (1957). Note that the Fisher–Kenen analysis proceeds on the basis of $k \times n$ distribution matrices in which there are k goods and n people (with no direct use of information on prices), whereas the system used here relates such judgements to the money income n-vector and the price k-vector (without information on the interpersonal distribution of the physical commodities).

glaring that the ranking may be extremely easy, but this may not be so in other cases. It may be, therefore, advisable to take B to be a quasi-ordering rather than assume it to be an ordering.

A very serious difficulty lies in the fact that frequently welfare judgements may have to be made without any clear knowledge of the relevant price vectors. Typically, distributional judgements are made with only a modicum of knowledge about the prices that are ruling. Would it be correct to assume that such judgements made in the absence of precise information on prices must be completely arbitrary? This need not be the case at all. Often we may have a reasonably clear idea of the range within which the vector of prices may lie even though we may not know the exact price vector, and we may commit ourselves only to those judgements which would hold for *all* price vectors within that range.

Formally, let Δ be the set of possible price vectors and define the binary relation J as:

$$yJx \text{ if and only if}$$
$$[\text{For all } p^1, p^2 \text{ in } \Delta: (y, p^1)\ B\ (x, p^2)] \tag{3.16}$$

The following result is of some interest.

Theorem 3.4

If B is transitive, then so is J. If B is an ordering, then Δ being a unit set is sufficient but not necessary for J to be an ordering.

The result is quite straightforward. If for all p^1, p^2, p^3, p^4 in Δ, $(y, p^1)\ B\ (x, p^2)$ and $(z, p^3)\ B\ (y, p^4)$, then obviously for all p^1, p^2, in Δ, $(z, p^1)\ B\ (x, p^2)$, given the transitivity of B. So J is also transitive. If Δ is a unit set, then J must also be reflexive. Further, since B is complete if it is an ordering, clearly with only one p in Δ, J must be complete too. On the other hand, let there be only there alternatives (x, y, z), and say $(y, p^1)\ B\ (x, p^2)$ and $(z, p^3)\ B\ (y, p^4)$, for all p^1, p^2, p^3, p^4 in Δ. Thus, (z, y, x) is a J-ordering despite Δ not being necessarily a unit set.

Note that J is not necessarily reflexive and therefore may not be a quasi-ordering. Two identical money-income distributions may indeed not be socially as good as each other if prices differ. In fact, two identical distributions of money income would typically be ranked differently depending on the prices ruling in each case, and we may not be able to say much without knowing the prices. On the other hand, if one distribution involves a much higher extent of concentration than another, it may be possible to be sure that its welfare value would be less than that of the other within a fairly wide range of price variations.

The transitivity of J is an interesting property. If we define B to be asymmetric, as we well might, then J would be a 'strict partial ordering'. How extensive J would be would, of course, depend on the range defined by the set Δ of possible price vectors as well as on the relation B. The more complete the price information and the more extensive B is, the more extensive would J tend to be. What is most important to recognize is that the choice is not of all-or-none type, and some systematic welfare judgements on money-income distributions with incomplete price information may be still possible.

The real trouble lies with defining the *same* level of real income, separating out the problem of distribution from that of the size of the total income. This is an old problem and has been much discussed in the literature of welfare economics.[18] If one considers distributions of the same level of *money* income with different prices, it is tempting to distinguish between two elements in the welfare variation, viz., (i) that due to differences in the aggregate real income, and (ii) that due to differences in the distribution of that income. There is, however, no uniquely appropriate method of doing the split up, and the arbitrariness of the pure distribution problem is just the 'dual' of the much-studied arbitrariness of real-income comparisons.

Nevertheless, for any *given* definition of real income, we can apply distributional judgements within that framework. If x and y are two money-income distributions that are judged to

[18] See Samuelson (1950).

have the same total real income, then yBx can be identified as reflecting that y is a better distribution than x. If we are uncertain of the price vector, we can relate distributional judgements to the strict partial ordering J.[19] Any such judgement would be conditional on a particular method of real-income comparison, but it can scarcely be otherwise. The problem of distribution of a 'given' real income clearly must depend on the definition of real income.

Variations of mean income

I turn now to the problem of the variation of mean income. The usual descriptive measures of inequality—such as the range E, the relative mean deviation M, the coefficient of variation C, the Gini coefficient G, or the standard deviation of logarithms H—all concentrate on *relative* variations of income. Among the descriptive measures studied in Chapter 2 only the variance V was not mean-independent. However, the normative measures presented in that chapter all operated on the *same* mean income, and measures D, A, and N, were all cast in this narrow framework. Can these measures be made mean-independent? And should we wish to do this?

Because of its dependence on the convention of utility scaling, Dalton's measure D may be thought to be inferior to Atkinson's A. The measure N is, however, more general, since it is not based on the restrictive assumption of additive separability. But, for the same reason, its general properties are more difficult to specify than those of A. In fact, the condition that the measure N be independent of mean income and be dependent only on the *relative* distribution of income does not yield any very obvious pattern, whereas the same condition when imposed on the additive structure of Atkinson's measure A with identical individual U functions immediately yields a straightforward pattern for the welfare function.

[19] It might be wondered whether there is not a contradiction in assuming that prices are known for determining whether the real income is the same but not for using welfare judgements B. But the prices relevant for the two exercises are not the same, and real-income comparisons are partly a matter of pure convention whereas welfare judgements require very specific price information for each year.

It is easily checked that A will be independent of the level of mean income if and only if the individual utility function U takes the following form:[20]

$$U(y_i) = k_1 + (k_2/\alpha)(y_i)^\alpha, \qquad (3.17)$$

where k_1 and k_2 are two constants, and the elasticity α must be less than or equal to 1 for the concavity of the U function. This constant elasticity form is obligatory if social welfare takes the utilitarian shape of being additive on identical U functions. While the case is rather restrictive, as Atkinson (1970) notes, the group welfare function is still capable of varying from the one extreme of being linear on individual incomes, thereby ranking distributions solely according to total income (for $\alpha = 1$), to the other extreme of ranking distributions solely according to the minimum income level $\min_i\{y_i\}$ and ignoring the other incomes (for $\alpha = -\infty$).[21]

Despite this pleasing robustness, the fact remains that (3.17) is a highly restrictive form. It also corresponds to the very limited case of the additive group welfare function, the weakness of which I have tried to discuss earlier. What is really restrictive, however, is the condition itself, viz., the requirement that the inequality measure should be independent of the mean income level. One can argue that for low income levels the inequality measures should take much sharper note of the same degree of *relative* variation on the ground that inequality pinches most when people are closer to starvation. On the other side, I have heard it argued that equality is a 'luxury' that only a rich economy can 'afford', and while I cannot pretend to understand fully this point of view, I am impressed by the number of people who seem to be prepared to advocate such a position. Though the considerations run in opposite directions, that in itself is no justification for making

[20] See Atkinson (1970), p. 251. This result is, as Atkinson notes, essentially a reinterpretation of a result derived by Pratt (1964) and Arrow (1965) for the theory of risk bearing. For the case of $\alpha = 0$, we have $U(y_i) = \log_e(y_i)$.

[21] The latter corresponds to the criterion of justice proposed by Rawls (1971).

the inequality measure *independent* of the level of mean income.

We are caught in a bit of a dilemma here. Making inequality measures independent of the mean income seems objectionable, but no alternative general assumption about the relationship of the mean income to these measures seems to be acceptable to all. Also, quantitative specification of the extent of the dependence on the mean income would bring in division even within a camp that may be united on the *direction* of the dependence and *only* on the direction.

Description and non-compulsive judgements

As discussed earlier, a possible alternative is to use mean-independent measures as *prima facie* but tentative measures of inequality, but to supplement them with other considerations that relate systematically to the level of mean income. This supplementation can be done in one of two ways. First, it may be possible to argue that while distribution x is more unequal than y according to some mean-independent measure, since y involves a lower mean income than x, maybe y represents more 'real' inequality. A second alternative is to be unambitious from the normative point of view as far as the measure itself is concerned, and to confine oneself to mean-independent inequality measures with a frank recognition that such measures may not have a high normative content. One can then argue that x may be more equal than y in the only sense in which the measurement is being made, but the relative welfare *impact* of inequality could be greater for y than for x since y corresponds to a lower mean income. The difference between this position and the first lies precisely in the extent to which inequality measures are themselves expected to reflect the relevant normative values rather than being positive measures in terms of which normative judgements may be conveniently expressed. I have discussed this distinction earlier.

Of course, even in this limited form a measure of relative inequality derived from some welfare considerations (though being independent of mean income) would, naturally, have a normative content. This would reflect a 'non-compulsive' judgement implying a *prima facie* evaluation of welfare which should be interpreted as entailing a recommendation unless other arguments can be summoned against such a recommendation. In having this qualified normative aspect combined with descriptive features, such a measure seems also to be reasonably close to the non-technical concept of inequality as employed in normal communication.

Intersection quasi-orderings

Turning now to the descriptive side, it is significant to note that the alternative indicators tend to involve some conflicts and some corroboration of each other. We can sort out the picture of partial correspondence by taking the intersection of the set of chosen measures. When there are k criteria, C^j for $j = 1, \ldots, k$, each yielding a complete ordering, we can define their intersection Q as:

$$yQx \text{ if and only if } [\text{for all } j = 1, \ldots, k: yC^jx]. \quad (3.18)$$

Theorem 3.5

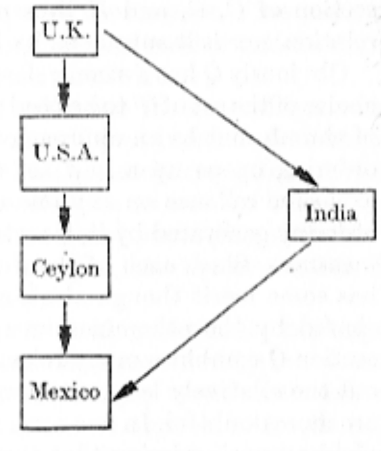
Q is a quasi-ordering.

This is readily checked, since the reflexivity of Q is not in doubt given the reflexivity of each C^j, and transitivity of Q follows from the fact that if zC^jy and yC^jx for all j, then zC^jx for all j, given the transitivity property of each C^j.

Such an intersection quasi-ordering has the advantage of avoiding exclusive reliance on any particular measure and on the complete ordering generated by it which reflects its arbitrary features. On the other hand, Q might be rather severely incomplete and precisely how incomplete would depend on the extent to which the various C^j measures conflict. Some comparisons would yield definite results while others would not. The point may be illustrated by the quasi-ordering of income distributions in five countries, viz., the U.K., the U.S.A., Mexico, Ceylon, and India shown in Diagram 3.5, based on three measures, viz., the Gini coefficient, the coefficient of variation, and the standard deviation of logarithms.[22]

[22] The data are taken from Table 1 in Atkinson (1970).

The U.K., the U.S.A., Ceylon and Mexico can be put on a simple ordering in terms of Q, but India brings out the incompleteness in being non-comparable with the U.S.A. and Ceylon, though it has a more unequal distribution than the U.K. and a less unequal one than Mexico. In particular, India seems to have a lower Gini coefficient and a lower standard deviation



Quasi-ordering based on C, G and H

DIAGRAM 3.5

of logarithms than Ceylon but a higher coefficient of variation, and similarly it has a higher Gini coefficient and a higher coefficient of variation than the U.S.A. but a lower standard deviation of logarithms.

Would Q be more extensive than the Lorenz relation L or the weak version of it, R? Or less so? Either is possible. Obviously the three criteria C, G, and H together cannot guarantee that the Lorenz relation will go the same way. This is obvious since the Lorenz relation requires that n inequalities be satisfied when n is the population size.[23] The three rankings C, G and H will yield only three inequalities, and that can hardly cover it.

[23] See inequalities (3.6) in the proof of Theorem 3.1.

On the other hand, the Lorenz relation cannot subsume the set of three descriptive criteria, since they do not all have the required concavity properties. The coefficient of variation C is concave, but the Gini coefficient G is not strictly quasi-concave, but just quasi-concave. G does not, however, conflict with the Lorenz relation, but the standard deviation of logarithms H does, as can be readily checked.[24] Q based on the intersection of C, G, and H thus neither subsumes the Lorenz relation, nor is it subsumed by it.

Obviously Q has a strong element of arbitrariness, since the choice of the set of C^i to be used would reflect some kind of rule of thumb, but as an approach that of an intersection quasi-ordering opens up a new set of possibilities. In eschewing exclusive reliance on any one measure and on the complete ordering generated by it, Q restrains the arbitrariness of such measures. Since each of the measures chosen for deriving Q has some merit though also some deficiencies which are not shared by the other measures (see Chapter 2), their intersection Q combines many relevant features and helps to sort out the relatively less controversial rankings from those that are more doubtful. In fact even the three measures mentioned yield a quasi-ordering that turns out to have quite a bit of cutting power; it is a matter of some empirical interest—though not of any great analytical significance—that the Q generated by the intersection of these three criteria for the twelve countries for which Atkinson (1970) presents data remains *completely unchanged* when the set of C^i is expanded to include Atkinson's three normative measures ('equally distributed equivalent') as well.[25]

[24] The welfare function corresponding to H is not strictly S-concave, which is the necessary and sufficient condition for the Lorenz ranking to be a sub-relation of H. On the other hand, G *is* strictly S-concave.

[25] However, the quasi-ordering Q generated by the three normative measures shrinks when Q is made to take note of the three descriptive measures as well. That is, the intersection of the descriptive measures is a sub-relation of the intersection of Atkinson's three normative measures, but not vice versa. Note, however, that the three normative measures the values of which are given by Atkinson are all of the particular form of (3.17) and differ only in values of α. If a wider class of normative measures are considered, their intersection would tend to be smaller.

A less uptight framework

I have tried to argue in favour of weakening the inequality measures in more than one sense. First of all, the mixture of partly descriptive and partly normative considerations weakens the purity of an inequality index. A purely descriptive measure lacks motivation, while a purely normative measure seems to miss important features of the concept of inequality. Some alternative ways of combining normative and descriptive considerations have been considered.

Second, even as normative indicators the inequality measures are best viewed as 'non-compulsive' judgements recommending something but not with absolutely compelling force. This has implications in terms of the treatment of inequality rankings as *prima facie* arguments and permitting situation-specific considerations to be brought into the evaluation if such supplementation is needed.

Third, a number of reasons for taking inequality rankings as quasi-orderings rather than as complete orderings have been suggested. One reason is the uncertainty about the welfare function to be used in the normative approach. Another is uncertainty about prices and real income, as well as the general difficulties of forming distributional judgements in a multi-commodity world. If we wish to incorporate dependence on the mean income into the measure itself (this is not, of course, the only way of handling the problem), then that too would push us in the direction of incompleteness, requiring abstention from ranking inequalities when the mean income differences are large and significant.

Even the descriptive measures, each of which yields a complete ordering, point collectively towards incomplete quasi-orderings. The intersection of these rankings tends to separate out relatively simpler comparisons from the more complex ones.

The very notion of inequality seems to have this quasi-ordering framework. The concept is not geared to making fine distinctions and comes into its own with sharper contrasts. This is also suggested by the relation—discussed earlier—

between the idea of inequality and the inclination to protest and rebel.

Treating inequality as a quasi-ordering has much to be commended from the normative as well as the descriptive point of view. I would suspect that the empirical work in this field would gain in meaningfulness if the all-or-none approach of traditional theory were abandoned and the arbitrariness of the usual complete orderings avoided. The glib man who can make inequality comparisons perfectly between every pair of distributions and the wise guy who finds all such comparisons 'arbitrary' both seem to miss essential aspects of the concept of inequality.

4

Work, Needs, and Inequality

IN this last chapter I should like to go into some rather broader issues concerning economic inequality. Inequality is sometimes viewed in relative terms, viz., as a departure from some notion of appropriate distribution. There are essentially two rival notions of the 'right' distribution of income, based respectively on needs and desert. It is easy to recognize the contrast between arguments of the kind: 'A should get more income than B since his needs are greater', and those of the type: 'A should get more income than B since he has done more work and deserves a higher reward'. Inequality can, therefore, be viewed not merely as a measure of dispersion but also as a measure of the difference between the actual distribution of income on the one hand and *either* (i) distribution according to needs, *or* (ii) that according to some concept of desert. I shall discuss each of the two approaches in turn.

Needs and welfare

The concept of relative needs is, of course, closely connected with the pattern of individual welfare functions and the type of interpersonal considerations that were discussed in Chapter 1. However, there are some pitfalls in doing the translation from needs to welfare. It might, for example, appear that a more needy man should get more out of a given income and, therefore, his welfare from a given level of income y should be *higher* than that of a person with less needs. But a little reflection should make clear that the inequality should point the other way. Clearly one would prefer to be a person with income y and less needs (e.g., normal health) than a person with income y and more needs (e.g., a malfunctioning kidney); and in terms of the framework of interpersonal comparisons outlined in

Chapter 1 (viz., in terms of $\tilde{R}$), this means that the first person would have a higher level of welfare than the second.

The Weak Equity Axiom and other equity considerations discussed in Chapter 1 would recommend a higher share of total income going to a person with a uniformly lower welfare function, i.e., to a person with greater needs. It might be considered how such needs could be determined and whether, in practice, greater needs could really serve as a basis for receiving a higher share of income. Can one really identify greater needs in any convincing way?

The problem of assessing relative needs is indeed a very serious one, and there can be hard problems of decidability. There is, however, the danger of falling prey to a kind of nihilism that characterizes much of normative economics and which we have been battling against in other contexts in the earlier chapters. This takes the form of noting, quite legitimately, a difficulty of some sort, and then constructing from it a picture of total disaster. Sure enough, greater needs are hard to identify sometimes, but they are quite clear at other times. For anyone making the judgement, the test is to ask oneself: Would you prefer to be person A with income y or person B with income y? An illustration may make the point clearer.

National Health service versus medical insurances

The rationale of medical facilities as a public service has been the subject of some debate in economics. The failure of the market to provide insurance against medical uncertainties has been illuminatingly analysed by Arrow (1963), but as Arrow himself points out, if the insurance markets were perfectly competitive, 'those in groups of higher incidences of illness should pay higher premiums'.[1] This means that those with a higher incidence of illness would end up with less income *net* of insurance premiums. This is, of course, precisely what a national health service run independently of market profitability can avoid. But what is the rationale of avoiding it? Precisely the needs principle which we have been examining. An ill person has identifiably greater needs, and by spending more money on him the society would give him a greater effective income, which is precisely in line with the Weak Equity Axiom discussed in Chapter 1.

[1] Arrow (1963), p. 205.

While this is perhaps not the occasion to comment on the relative merits of giving ill people cash subsidies against those of providing them with free medical services, I would never the less comment briefly on an aspect of the problem which seems to me to touch on the question of decidability of relative needs. I do not wish to go into the organizational advantages of providing medical services through a national health network and the possible economies of large scale involved in this, but it is pertinent to note that the provision of cash subsidies opens up greater possibilities of abuse through pretensions of greater needs, thereby bedevilling the problem of decidability. When medical services are provided in kind, the link-up with needs is more direct and the practical problem of identifying needs is to that extent reduced. The national health service has a built-in system of attempting to match payments to needs, and this is of obvious relevance to any comparison of the merits of the two systems of compensation.

Non-income determinants of welfare

In taking a group welfare function of the form $F(y_1, \ldots, y_n)$, non-income considerations of relevance to social welfare (e.g., sweat of work) can be brought into the picture only through the shape of the function F. Taking the individualistic case, in which social welfare is a function of individual welfare levels, $W(U_1, \ldots, U_n)$, if it is further specified that each U_i is a function of income only, $U_i(y_i)$, or more generally $U_i(y_1, \ldots, y_n)$, once again there will, of course, be no way of bringing considerations like variable sweat except through the form of the functions U_i. This functional variation would, formally, reflect variable 'needs' for income of the persons given by their non-income characteristics.

For example, consider two income distributions x and y, with identical total, over a collection of n people who are symmetrical in all respects except that person 1 works in a nasty coal mine and has tougher working conditions than persons 3 to n, while person 2 works under more pleasant working conditions than these other persons. Let x be a completely equal distribution, whereas y gives more income to person 1 and less to person 2 than the rest. We might conceivably decide to prefer y to x on the grounds that for the same level of income person 1's welfare would be less, and person 2's greater, than the welfare of everybody else. The Weak Equity Axiom would recommend choosing some y (not necessarily *every* arbitrarily picked y) satisfying these inequalities, and if y is preferred to x on that ground, the way of characterizing it would be in terms of person 1 having greater need for income given his tough work conditions and person 2 having less need given his favourable non-income situation.[2] In terms of our model of interpersonal comparisons, preferring to be person 2 than person 1 at the same level of income, is equivalent to asserting that 2 has greater welfare than 1 at the same level of income, and that in its turn is taken to be equivalent to the first person having greater needs than the second.[3]

The assumption of symmetry in the *evaluation* of income distributions may, therefore, have to be rejected not merely on grounds of inherent differences in needs (e.g. some people being chronically ill or crippled), but also because of differences in non-income characteristics (e.g. particular working conditions). In a system geared to ranking distributions of income as such, considerations of this type must come under the

[2] An alternative way of handling this particular problem is to look at the distribution not merely of income but of utilities defined as functions of income and work efforts. See, for example, Kolm's 'leisurely equivalent income' (Kolm, 1969, pp. 181–2). But, of course, there would also be other differences of relevance to the distribution problem, e.g., location characteristics, cultural propensities, etc. Kolm (1969) provides an interesting discussion of the distribution problem in a highly general setting.

[3] Note that the same would hold if person 2 had greater wealth than person 1, i.e., it would be judged that he had less 'need' for income, other things given.

broad hat of variations of needs, and would take the form of differences in welfare functions defined on the income levels.

Variations of unidentifiable characteristics

The historic controversy on the difficulty of interpersonal comparisons of welfare, and therefore of needs, took place not in the context of these identifiable differences of need characteristics (such as being a cripple or having terrible working conditions), but in that of alleged interpersonal differences which were not necessarily identifiable in objective terms. In his classic article on interpersonal comparisons Robbins (1938) made use of a story attributed to Sir Henry Maine in which a Brahmin, confronted with a Benthamite, kept insisting: 'I am ten times as capable of happiness as that untouchable over there.' Reflections on this argument led Robbins to the conclusion: 'I could not escape the conviction that, if I chose to regard men as equally capable of satisfaction and he to regard them as differing according to a hierarchical schedule, the difference between us was not one which could be resolved by the same methods of demonstration as were available in other fields of social judgement.'[4]

There are two distinct elements in this line of argument. First, there is the question of the inherent impossibility of interpersonal comparisons. In support of his position Robbins quotes Jevons as saying: 'I see no means whereby such comparison can be accomplished. Every mind is inscrutable to every other mind and no common denominator of feeling is possible.'[5] I do not wish to go here into the somewhat solipsistic implications of this position, nor into the undoubted fact that every mind is *not* inscrutable to every other (not even oriental minds, as Maine and Robbins bear out), nor into the relevance of the 'common humanity' of men for the evaluation of social arrangements, illuminatingly analysed by Bernard

[4] Robbins (1938), p. 636.

[5] Robbins (1938), p. 637.

Williams.[6] For our purpose it is sufficient to note that we have been interpreting interpersonal comparisons of welfare in terms of choices of being in the position of one person rather than that of another. Under this system 'the common denominator of feeling' is not far to seek, and systematic thinking about it seems perfectly possible.[7]

The second element in the argument arises not from the alleged impossibility of making interpersonal comparisons but from the possibility that it might in fact be held that Maine's Brahmin was indeed ten times as capable of happiness as the other man. Two questions arise here, viz., (i) why so? and (ii) what then? Taking the second question first, if Maine's Brahmin were right and if his statement were interpreted to mean that he indeed had ten times as much welfare as the untouchable for any given income level, then the Weak Equity Axiom would immediately recommend that the Brahmin be given *less* income than the untouchable! Maine's Brahmin got away with his argument (if argument it was), only because he was facing a Benthamite and only because Robbins paid the ultimate compliment to his utilitarian adversaries by being

[6] 'That all men are human is, if a tautology, a useful one, serving as a reminder that those who belong anatomically to the species *homo sapiens*, and can speak a language, use tools, live in societies, can interbreed despite racial differences, etc., are also alike in certain other respects more likely to be forgotten. These respects are notably the capacity to feel pain, both from immediate physical causes and from various situations represented in perception and in thought; and the capacity to feel affection for others, and the consequences of this, connected with the frustration of this affection, loss of its object, etc. The assertion that men are alike in the possession of these characteristics is, while indisputable and (it may be) even necessarily true, not trivial. For it is certain that there are political and social arrangements that systematically neglect these characteristics in the case of some groups of men, while being fully aware of them in the case of others; that is to say, they treat certain men as though they did not possess these characteristics, and neglect moral claims that arise from these characteristics and which would be admitted to arise from them.' (Williams, 1962, p. 112.)

[7] I am not sure that 'the same methods of demonstration' does not apply here 'as were available *in other fields of social judgement*' (Robbins 1938, p. 636; italics mine). The distinction that is being made is not particularly clear especially since Robbins admits into his framework of 'scientific foundations' both 'observation' and 'introspection' (pp. 637 and 640).

able to think of no way of handling individual utilities except by *adding* them.

Probabilistic egalitarianism

But even *within* the utilitarian framework and even after noting that people may indeed have different utility functions, it can be asked: Why is it more likely that the Brahmin has a greater capacity for satisfaction than the untouchable? What if we assume that it is as likely that this is the case as that the opposite holds? What then? It is to this question that Abba Lerner (1944) had addressed himself in dealing with distributional problems in a socialist economy. Lerner's answer for distributing a given total income was that the right solution in such a situation was to divide it equally. Since doubts have been raised from time to time as to what precisely Lerner's theorem amounts to and whether it is valid,[8] a somewhat formal presentation of the result is called for. This is not very difficult to give, and I have in fact presented such a formulation elsewhere (Sen 1969). What is much more important is to rescue Lerner's result from its reliance on the utilitarian framework which we have found to be objectionable (see Chapter 1), and to look for a theorem that would be valid not only for the utilitarian case but also for others. Such a generalization is indeed possible.[9]

Assumption 4.1 (Total Income Fixity): There is a fixed income y^* to be divided among n individuals, i.e., $y_1 + \ldots + y_n = y^*$.

Assumption 4.2 (Concavity of the Group Welfare Function): Social welfare W, a symmetric and increasing function of individual welfare levels $W(U_1, \ldots, U_n)$, is concave.

Assumption 4.3 (Concavity of the Individual Welfare

[8] See Friedman (1947), Samuelson (1964), Breit and Culbertson, Jr. (1970).

[9] We are sticking to n possible individual welfare functions. It is easy to drop this requirement (see Sen 1969), but the intuitive aspect of the Lerner problem is caught rather well by the case in which there are n persons and n individual welfare functions but it is not known *who* has *which* function.

Functions): There are n individual welfare functions $U^1(y), \ldots, U^n(y)$, and each of them is concave.

Assumption 4.4 (*Equi-probability*): If $p^j{}_i$ is the probability that person i has the welfare function U^j, then for all j, $p^j{}_i = p^j{}_h$, for all individuals i, h.

Theorem 4.1

Given Assumptions 1, 2, 3, and 4, the mathematical expectation of social welfare is maximized by an equal division of income.

Thanks to the symmetry of W, we can define a group welfare function $W = F(y^1, \ldots, y^n)$, in which y^j is the income going to the person with the j-th welfare function U^j. For any income distribution $(y_1, \ldots, y_n)$, any reordering of it $(y^1, \ldots, y^n)$ essentially reflects a particular assignment of individual welfare functions to the persons in the group. For any distribution vector y, there are $n!$ such reorderings $\tilde{y}(k)$, $k = 1, \ldots, n!$, and corresponding to each k, there is a specific value of social welfare given by $F(\tilde{y}(k))$. Since Assumption 4.4 implies that each of the possibilities are exactly equally likely, the mathematical expectation E of social welfare is given by:

$$E(y) = \frac{1}{n!} \sum_{k=1}^{n!} F(\tilde{y}(k)). \tag{4.1}$$

If x is an equal-distribution vector, i.e., $x_1 = \ldots = x_n$, then clearly:

$$E(x) = F(x) \tag{4.2}$$

By Assumption 4.1 it is obvious that:

$$x = \frac{1}{n!} \sum_{k=1}^{n!} \tilde{y}(k) \tag{4.3}$$

By Assumptions 4.2 and 4.3, $F(\,.\,)$ is a concave function, and therefore from equations (4.1), (4.2), and (4.3), it must be the case that:

$$E(y) \leq E(x) \tag{4.4}$$

Since (4.4) holds for all y, evidently Theorem 4.1 must be true.

Note that this result is not subject to the criticism that Milton Friedman (1947) made of Lerner's welfare function by considering the case in which there is no ignorance:

> Suppose, further, that it is discovered . . . that a hundred persons in the United States are enormously more efficient pleasure machines than any others, so that each of these would have to be given an income ten thousand times as large as the income of the next most efficient pleasure machine in order to maximize aggregate utility. Would Lerner be willing to accept the resulting division of income as optimum . . . ?[10]

Happily, Lerner does not have to express such a willingness. In fact, he can even confine himself to the class of concave group welfare functions satisfying the Weak Equity Axiom, which would rule out the possibility that Friedman suggests and in fact ensure that the more efficient pleasure machine would be handed out *less* income. Even then the right distribution in a state of ignorance would be the equal one. Lerner's probabilistic egalitarianism need not be based on the utilitarian framework at all (though it does happen to hold for that case as well).[11]

Maximin egalitarianism

The equi-probability assumption has been subjected to some severe criticism. It can indeed be argued that not to be sure who has which utility function is not the same thing as assuming that every possible assignment is equally likely. Perhaps a more interesting assumption than Assumption 4.4 is the following.

*Assumption 4.4** (*Shared Set of Welfare Functions*): For any person i and any utility function j, it is possible that i has j.

Since nothing is now said about probability, the mathematical expectation of social welfare cannot any more be defined. But there are other criteria one can use, and in

[10] Friedman (1947), pp. 310–11.

[11] It may be wondered whether maximization of the mathematical expectation of social welfare would not be senseless in the non-utilitarian case. But this is not so. The simplest case to consider is a non-utilitarian F which is still additively separable, e.g., taking strictly concave transforms of people's utilities and then adding them.

particular the 'maximin' policy of maximizing the minimal level of social welfare. To guarantee that the minimum exists for each assignment, we need some additional assumption, and this we do with a simple requirement (though it is, in fact, unnecessarily strong).

Assumption 5 (*Bounded Individual Welfare Functions*): Each individual welfare function U^i is bounded from below.

What kind of a distribution policy would the 'maximin' strategy recommend? Once again an equal distribution, as was shown for the utilitarian case in Sen (1969), but the result is easily generalized for all concave group welfare functions (indeed also for all quasi-concave functions as well).

Theorem 4.2

Given Assumptions 1, 2, 3, 4*, and 5, the maximin strategy for social welfare is to distribute income equally.

Consider the set of all $\bar{y}(k)$ for $k = 1, \ldots, n!$. Since F is quasi-concave and x is a weighted average of all such $\bar{y}(k)$, clearly:

$$F(x) \geq \text{Min}_k F(\bar{y}(k)). \qquad (4.5)$$

And this establishes the theorem since, x being an equal division, $F(x)$ is invariant with respect to interpersonal permutations of individual welfare functions.

Thus not only is the equal distribution an optimal policy to be followed if the mathematical expectation of social welfare is to be maximized in a situation of ignorance under the assumption of equi-probability, it is optimal also for the 'maximin' strategy completely independently of the relative probability distributions.[12] Since there are people who seem

[12] It is important to avoid confusion between the 'maximin' criterion of Rawls (1971), in which the level of welfare of the worst-off individual is maximized with no uncertainty about who has which welfare function, and the 'maximin' strategy referred to in Theorem 4.2, in which the minimal level of social welfare, which can be any concave function of individual welfares, is maximized in a situation of ignorance as to who has which welfare function. Since Rawls's 'maximin' rule yields a concave group welfare function, it is covered by Theorems 4.1 and 4.2, and the results apply to the 'maximin' conception of social welfare and to the use of the 'maximin' strategy within that conception. The maximin–maximin policy is still an equal distribution.

to like paradoxes, I leave it to them to chew over the idea that a 'conservative' policy like the 'maximin' yields a 'radical' conclusion like absolute equality in income distribution, but I fear I cannot recommend it as a very juicy paradox.

It appears that egalitarianism may be optimal under ignorance about relative needs (and therefore about individual welfare functions), and not merely under perfect certainty with the same welfare function being shared by all. Results of the type presented in Theorems 4.1 and 4.2 have to be contrasted with our observations on *identified* differences of welfare functions, e.g., the case of the cripple. Being sure about unequal needs would certainly push us in the direction of an unequal division of income corresponding to relative needs, thanks to the Weak Equity Axiom and similar requirements, but these axioms do not seem to provide a justification for departing from equality of incomes when we are not sure about relative needs. The two generalizations presented here of Lerner's pioneering result in this field permit us to combine Lerner's egalitarian conclusion with adherence to the Weak Equity Axiom and other requirements of equity.

Needs principle versus the works principle

I referred earlier to the contrast between the principle of distribution according to needs and that of distribution according to desert. The usual interpretation of desert is in terms of some conception of value of work done. The Marxian notion of 'exploitation' is based on the concept of 'surplus value', viz., the difference between the value added and the wages paid, and the ratio of the surplus value to the wages bill is taken to be the rate of exploitation. As a general approach this certainly falls in the category of being desert-based rather than needs-based.

While exploitation has played an important part in Marxian economics, it would be a mistake to think that deserts took priority over needs in the Marxian analysis of distribution, or that Marx was not clear on the distinction. In fact he made the distinction very sharply and accepted the ultimate superiority of the needs principle. In his *Critique of the Gotha Programme* of 1875 he took the German Workers' Party very severely to task for confusing the two principles. Pointing out the contradiction displayed in the *Gotha Programme* between the principle of the worker's right to get 'the undiminished proceeds of labour' and that of giving 'equal right to all members of society' to the output of the society, Marx went on to associate the two principles with two different phases of socialism. Since this analysis has been the starting point of many debates in the socialist literature, and since—as I would argue later—the same set of issues recurs systematically in the technical literature on optimal allocation of resources, I take the liberty of quoting Marx in some detail:

What we have to deal with here is a communist society, not as it has *developed* on its own foundations, but, on the contrary, just as it *emerges* from capitalist society; which is thus in every respect, economically, morally and intellectually, still stamped with the birthmarks of the old society from whose womb it emerges. Accordingly, the individual producer receives back from society—after the deductions have been made—exactly what he gives to it. . . . He receives a certificate from society that he has furnished such and such an amount of labour (after deducting his labour from the common funds), and with this certificate he draws from the social stock of means of consumption as much as costs the same amount of labour. . . .

Hence, *equal right* here is still in principle—*bourgeois right*, although principle and practice are no longer at loggerheads, while the exchange of equivalents in commodity exchange only exists *on the average* and not in the individual case.

In spite of this advance, this *equal right* is still constantly stigmatized by a bourgeois limitation. The right of the producers is *proportional* to the labour they supply; the equality consists in the fact that measurement is made with an *equal standard*, labour.

But one man is superior to another physically or mentally and so supplies more labour in the same time, or can labour for a longer time; and labour, to serve as a measure, must be defined by its duration or intensity, otherwise it ceases to be a standard of measurement. This *equal* right is an unequal right for unequal labour. It recognizes no class differences, because everyone is only a worker like everyone else; but it tacitly recognizes unequal individual endowment and thus productive capacity as natural privileges. *It is, therefore, a right of inequality, in its content, like every right.* Right by its very nature can consist only in the application of an equal standard; but unequal individuals (and they would not be different individuals if they were not unequal) are measurable only by an equal standard in so far as they are brought under an equal point of view, are taken from one *definite* side only, for instance, in the present case, are regarded *only as workers*, and nothing more is seen in them, everything else being ignored. . . .

But these defects are inevitable in the first phase of communist society as it is when it has just emerged after prolonged birth pangs from capitalist society. . . .

In a higher phase of communist society, after the enslaving subordination of the individual to the division of labour, and therewith also the antithesis between mental and physical labour has vanished; after labour has become not only a means of life but life's prime want; after the productive forces have also increased with the all-round development of the individual, and all the springs of cooperative wealth flow more abundantly—only then can the narrow horizon of bourgeois right be crossed in its entirety and society inscribe in its banners: From each according to his ability, to each according to his needs![13]

The two principles contrasted by Marx correspond to two ways of evaluating income distribution, and while the analysis of 'exploitation' deals with *desert*, the analysis of equality and crossing 'the narrow horizon of bourgeois right' relate to the concept of *needs*. The historical sequencing of the two phases of socialism with the two respective principles of distribution became the standard theory of socialist evolution and was not re-examined very critically until the recent Chinese attempts at building communes on the principle of needs at an early stage of socialism. The Chinese debate on the subject I shall comment on later, and I turn first to the relationship of all this to the academic literature on optimal allocation of resources.

Lange-Lerner systems

While much of the literature of optimal allocation is concerned with the achievement of only Pareto optimality (and

[13] Marx (1875), pp. 21–3.

therefore abstains from distributional questions), the two contributions on decentralized resource allocation that pioneered the study of the optimality aspects of price mechanism, viz., the works of Oscar Lange and Abba Lerner, were much concerned with the problem of right distribution. How did they face the conflict of the two principles outlined by Marx?

Lange (1936–37) noted the contrast between the two conditions involved in satisfying (i) distribution according to relative needs, i.e., 'the distribution has to be such that the same demand price offered by different consumers represents an equal urgency of needs', and (ii) the efficiency requirement 'to make the differences of the value of the marginal product of labour in the various occupations equal to the differences in the marginal disutility involved in their pursuit' (p. 101). But Lange thought that any contradiction between the two principles would be 'only apparent'. The former required an equal distribution of income if needs were equal, but so did the latter after taking note of the fact that 'the disutility of any occupation can be represented as opportunity cost'.

Lange seemed to be assuming equality of educational opportunity and training facilities which would explain much of the difference in productive abilities of different persons. As far as 'exceptional talents' were concerned, which formed a 'natural monopoly', he noted that they could be paid 'incomes which are far below the value of the marginal product of their services without affecting the supply of those services'.[14]

While this last point is of some importance—and we shall return to this question again later on in this chapter—there is little doubt that Lange was over-simplifying a complex picture. As Dobb (1933) had pointed out in an early critique of market socialism, there are questions of relative scarcity in any given market equilibrium and 'both costs and needs are precluded from receiving simultaneous expression in the same system of market valuations' (p. 37). Lange emphatically rejected Dobb's argument that these conditions were contradictory (p. 102), but Lange's market equilibrium seemed to

[14] Lange (1936–37) pp. 101–2. The last of these remarks was in response to a criticism by Dobb (1933).

assume (i) the absence of short-run scarcities, (ii) complete equalization of educational and training opportunities, including in the selection process, (iii) the absence of indivisibilities in the educational structure, and (iv) successful avoidance of payment of any 'rent' to natural talents. He also largely ignored the problem of incentives for intensive work effort, which had worried Marx.

Lerner (1944) was less optimistic and felt that 'the principle of equality would have to compromise with the principle of providing such incentives as would increase the total of income available to be divided' (p. 36). But where should the line of compromise be drawn? This is undoubtedly one of the more basic problems of socialist planning faced with the conflict between efficiency and equality.

Can taxes help in resolving the conflict? The question has cropped up in different forms repeatedly. In particular, it has been asked whether one can base pre-tax incomes in line with efficiency and post-tax incomes in line with needs. The answer is: Surely one can, but then why should the people in question take their decisions on efforts, leisure, etc., on the basis of their pre-tax incomes rather than on post-tax incomes? After all, pre-tax income is just a façade, and post-tax income is all that matters.[15] And then the conflict is back again—now related exclusively to post-tax incomes.

This recognition led to a search for a 'non distorting' tax. Is there such an animal?[16] In principle it seemed that 'lump-sum taxes' could do the trick. A lump-sum tax is unrelated to income, work, expenditure, consumption, saving, or anything else that a person can vary. By construction, therefore, lump-sum taxes cannot 'distort' allocation. Is this a fable? To discuss this I begin with a slight detour, viz., what goes wrong with the income tax.

[15] If pre-tax incomes have some 'prestige value', the picture will be more complex, but since men don't live by prestige alone, post-tax incomes will continue to influence individual decisions.

[16] This is an ancient issue in public finance, and the poll-tax has been much analysed. In the context of its use for socialist allocation and redistribution, see Samuelson (1947) and Dobb (1969) among others.

The income tax

The underlying problem can be explained in terms of a very simple model involving one commodity, i.e., homogeneous income. The following notation is used:

$y_i(t)$ = pre-tax income of person i under tax system t;
$y_i(0)$ = pre-tax income of person i in the special case of a no-tax system;
$y_i^*(t)$ = post-tax income of person i.

In the Lange–Lerner system, in the absence of externalities, increasing returns, and such things, $y_i(0)$ would correspond to the marginal productive contribution of each person's economic resources. A tax system may distort the person's decisions on work, leisure, etc., and the pre-tax income $y_i(t)$, in the presence of a system of taxes and subsidies, would represent a different equilibrium from that reflected in $y_i(0)$, because of the distortion of the reward system implicit in the taxes. On the other hand, $y_i^*(t)$, the income after taxes and subsidies, would presumably reflect the evaluation of needs and other distributional values used in the planning system.

Let w_i be the marginal income of worker i from a unit of effort, the hardship of which he evaluates as equivalent to α_i units of income at the margin. Let β_i be the value that worker i attaches to a unit of income going to others, measured in units of his own income. In the no-tax system, the worker will put in effort to the extent that:

$$w_i = \alpha_i \tag{4.6}$$

But with an income tax at the marginal rate of t per unit, $0 < t < 1$, he will equate:

$$w_i[(1 - t) + t\beta_i] = \alpha_i \tag{4.7}$$

(4.6) and (4.7) will be equivalent if and only if:

$$\textit{either } \alpha_i = 0, \textit{ or } \beta_i = 1 \tag{4.8}$$

These conditions correspond respectively (i) to the case in which the person does not mind expending effort and sweat,

and (ii) to the case where the person values the marginal income of others just as much as the marginal income of himself. Either of these conditions must be fulfilled for income tax to be non-distorting. But if $\alpha_i > 0$ and $\beta_i < 1$, then the income tax will distort allocation.

Lump-sum taxes

Can this problem be avoided? Are there taxes that will not have this distorting effect? First consider a relatively simple case in which a person's relative preference for income and leisure are not affected by his overall prosperity, though variations of the rate of remuneration for work would of course affect his work decisions.

Consider a fixed tax, t_i, on person i such that he must pay t_i no matter what else he does (works or not, eats a lot or a little, or anything else):

$$t_i = [\Sigma_i\ y_i(0)/n] - y_i(0) \tag{4.9}$$

Since the tax is fixed, the person cannot gain anything from varying his amount of work. Since his income-leisure preference is not affected by his level of prosperity, these lump-sum taxes leave everything completely unchanged as far as work and production are concerned. But the taxes (or subsidies since t_i can be positive, negative, or zero) take the system from one of distribution according to work to one of distribution according to needs.

The planners have to estimate the set of $y_i(0)$, which involves estimating the real capabilities of each person. There are two problems here, viz., (i) the cost of collecting the information, and (ii) the deliberate misinformation which person i might try to convey to the planners. The former can be quite serious, and it is of particular relevance to a system geared to achieving economy of information in the process of optimization. The decentralized system of the Lange–Lerner kind aims at reaching the optimum iteratively through trial and error with extreme parsimony in the transfer of detailed information. This problem is all the more serious when the assumption of invariance of income–leisure preference with respect to net prosperity is dropped. The non-distorting character of the lump-sum taxes still survives, but in calculating t_i from (4.9) one would have to interpret $y_i(0)$ valued not as it would be in the absence of all taxes, but after taking note of the impact of lump-sum taxes through being on a different part of the leisure–income indifference map. The marginal equilibrium given by (4.6) would still hold, and the lump-sum taxes would not interfere with the achievement of efficiency, but the calculations for (4.7) and (4.9) would be particularly complex, since α_i would depend on the level of income after the lump-sum tax.

The second problem would be an equally serious difficulty. It would be in the interest of each person to pretend to be less productive than he is and then to take things easy. By producing less oneself one reduces total output by a relatively small amount, and under egality the impact on one's net income is minute.

Hence with lump-sum taxes the distortion comes in not in the form of insufficient work effort *given* the tax system, but in that of giving wrong signals to the planners about one's productive ability, thereby influencing the tax system itself in one's favour. If person i can convince the planners that he is worthless and capable of no greater effort, then the value of t_i will be relatively smaller and he may be spared the necessity of exerting himself much. Such deliberate misinformation may bedevil the Lange–Lerner iterative procedure quite severely. Given a personal-gain oriented approach, this barrier is not easy to cross.

Work motivation

Underlying all this is precisely the problem of work motivation with which Marx was concerned. Marx saw no escape from it in the early phase of socialism in which the society and the people are 'economically, morally and intellectually, still stamped with the birthmarks of the old society from whose womb it emerges', and conceived of an ultimate solution to this problem in 'the all-round development of the individual', 'after labour has become not only a means of life but life's prime want'.[17] However, as we noted, he saw this only as a distant prospect.

The Soviet wage system reveals a concentration on work rewards and incentive payments,[18] which Marx had associated with the first phase of socialism. There are, of course, excep tions to this,[19] but the big point of departure can be associated with the Chinese attempt at communized agriculture with a deliberate move to achieve now what Marx had foreseen for the distant future. The Chinese experience on this is worth investigating in the context of the conflicting claims of the works principle and the needs principle.

During the so-called 'Great Leap Forward', which was launched in China in 1958, there was a strong move in the direction of non-material incentives, especially in agriculture. The proportion distributed according to work done was severely reduced, and the 'supply portion', which was distributed on some non-work criteria, including considerations of 'needs', was correspondingly raised. Sometimes even 80 to 90 per cent of the net product came to be distributed as the supply portion.[20]

In an economy like China there are several advantages in using a non-work basis of payments. First, as is well recognized in the literature on economic development, an important barrier to the utilization of surplus manpower is the wage system, which requires a prior supply of wage goods before underutilized labour can be mobilized.[21] A non-wage system would reduce the need for a *prior* surplus of wage goods, and labour could be rewarded by the fruits of its own output after the production lag. The Chinese were embarking on a vast programme of labour mobilization which included a remarkable amount of physical movement and migration.

[17] Marx (1875), pp. 21–3.

[18] See Dobb (1951), Nove (1961), Wiles (1962), Bergson (1964), and Ellman (1971).

[19] A system of free medical facilities, educational opportunities and social security, and subsidized housing and other services, does involve indirect use of the needs principle.

[20] See Hoffman (1964), (1967), and Riskin (1971).

[21] See Nurkse (1953), Robinson (1956), Sen (1964), and Marglin (1966).

Second, given the nature of the Chinese revolution and its predominant values, a system of 'material incentives' was regarded with considerable suspicion, and the Soviet concentration on an incentive system of rewards was the subject of much criticism. Thus philosophically and in terms of effective utilization of surplus manpower, the Chinese were poised for a move towards reliance on 'non-material incentives'. The 'leap' was taken in 1958.

During 1958–60 this experiment was carried out with much zeal along with other features that characterized the 'Leap Forward'. As is well known, the movement as a whole ran into several serious problems, but it is difficult to dissociate the difficulties generated by the use of non-material incentives from those caused by other features of the Leap Forward. It is certainly significant that as the movement came to an end the proportion distributed according to work was substantially raised and the use of the 'needs' principle was conceded to have been premature.[22] However, the emphasis on non-material incentives was not entirely abandoned and was partly revived later.[23] In fact, this feature of the substantial use of non-material incentives is recognized to be one of the remarkable aspects of the Chinese economy.

A game-theoretic presentation of the problem of work motivation

The problem of incentives that had bothered Marx was undoubtedly relevant to the Chinese experiment. It is, in fact, a basic question in collectivist allocation. The logic of the problem can be analysed in terms of some elementary games of the non-zero-sum variety. Interesting insights seem to come from contrasting games like the 'Prisoners' Dilemma'[24] with other games (like the 'Assurance Game'[25]) that differ from it

[22] Cf. 'But they [the communes] had been formed very hastily; the necessary psychological preparation had not everywhere been made, and some extreme ideas, such as abolishing private plots and distributing food according to needs rather than on work points, proved to be far ahead of the times. During the bad years reorganization took place and the extremist policies were abandoned.' (Joan Robinson 1969, p. 35.)

[23] Riskin (1971).

[24] See Luce and Raiffa (1958).

[25] See Sen (1967a), (1969a).

in some essential respects. While it is a trifle pompous to brandish little 'games' in analysing homely situations, I think there are substantial advantages in putting the analytical contrasts sharply to catch the precise motivational differences.

Suppose that a typical member of a cooperative considers two alternatives, viz., to work hard (I_1) and not to work hard (I_0). He may make two assumptions about others in the co-operative, viz., that they will work hard (R_1) or that they will not (R_0). Consider a system in which people are paid according to needs (and not work), whereas their main concern is with their own welfare. A typical ranking of alternatives may then take the form (in decreasing order of preference): I_0R_1, I_1R_1, I_0R_0, I_1R_0. By working hard oneself one adds very little to one's income since the principle of distribution is not work but needs, but there is still the hardship of toil. So given the actions of others, everyone may prefer not to work hard, i.e., prefer I_0 to I_1, no matter whether the others do R_0 or R_1. But at the same time they may each prefer everyone working hard to no one working hard, since the latter may be disastrous for all. In such a situation, however, guided by rational calculus everyone ends up not working hard, i.e., doing I_0, which is a strictly dominant strategy. But each would have preferred that all had worked harder. Individual rational calculations would seem to lead all to disaster.

This game—the Prisoners' Dilemma—has been much used in recent years to explain the rationale of an enforceable collusive solution in such fields as taxation, collective savings, etc.[26] However, since a collective contract with provision for enforcement may be extremely difficult to devise for labour efforts, the lesson to be drawn here has to be different. Work supervision to ensure adherence to a 'sincere effort' contract involves many problems,[27] and this is precisely where an

[26] See Baumol (1952), (1970), Sen (1961), (1967a), Marglin (1963), Ellman (1966).

[27] Work supervision of this kind may also bring out some of the most dislikeable features of 'alienation'—a major source of Marxian concern—'in the sense of labour "for somebody else", under the supervision and orders of somebody else'. (Mandel 1968, p. 680.)

incentive system of wages has an advantage.[28] The feasibility of using payments according to needs combined with vigorous supervision of work done is profoundly doubtful.

It is in this context that the question of cultural orientation of work motivation becomes crucially relevant, since the preference ordering in the Prisoners' Dilemma reflects a specific cultural pattern. Consider the following variation of the ranking of the alternatives: I_1R_1, I_0R_1, I_0R_0, I_1R_0. This produces a game ('the Assurance Game') in which each party would work hard (I_1) given the assurance that others would too (R_1) but would prefer not to put in the effort (I_0) if the others would not (R_0). The basic principle here is 'reciprocity', and this game can lead to an optimal solution in a situation of mutual confidence. If people's preferences are more 'socially conscious' in the sense of actually preferring to do the right thing whether or not others do the same, e.g. ranking the alternatives as I_1R_1, I_0R_1, I_1R_0, I_0R_0, everyone would automatically do his 'duty' and the question of supervision or even of confidence would not arise.

That the Prisoners' Dilemma could disappear if people had different preferences is true but hardly interesting. What is, however, quite significant is the fact that even if the people involved continued to have the same Prisoners' Dilemma type preferences, but behaved as if their preferences were as in the Assurance Game (or better still *as if* they had the 'socially conscious' preferences discussed above), they could be better off *even in terms of their true preferences*. This is precisely where the question of cultural orientation comes in, and it may provide a social case for encouraging values that reorient a person's choices and actions even if his personal welfare functions remain unaltered. In a sense, this is a matter of morality, and there are of course many other spheres of life as well in which a society throws up moral values that attempt to dissociate choice from individualistic rational calculus. Indeed this is a common phenomenon for 'homely virtues' like

[28] There are, however, allocational problems for a pure system of distribution according to work as well, on which see Ward (1958), Domar (1966), and Sen (1966).

honesty, keeping promises, etc., but what is important to recognize here is the relevance of all this to the problem of work motivation and therefore to income distribution.

Economic roots of the 'cultural revolution'

This dichotomy between choices on the one hand and preferences (and welfare) on the other has disturbing implications for the theory of 'revealed preference' and also has some bearing on theories of 'moral behaviour', neither of which I intend to pursue here.[29] What is of relevance here is the relation of all this to the conflict between the needs principle and the works principle, and in particular the light that this throws on the concentration on cultural reorientation that characterized China shortly after the end of the leap forward which had included the problem-ridden departure from payment according to work.

The economic roots of the Chinese 'cultural revolution' need careful attention. There were, of course, diverse forces involved in that movement, but certainly one strain in the discussion (and agitation) was closely related to the alternative principles of payment and to the question of work motivation. The official pronouncement on the subject explained that 'the aim of the Great Proletarian Cultural Revolution is to revolutionize people's ideology and as a consequence to achieve greater, faster, better and more economical results in all fields of work. . . . [it] is a powerful motive force for the development of social productive forces in our country'.[30] Using words reminiscent of those with which Marx had taken the *Gotha Programme* to task for ignoring the problem of work incentives in the early stages of the socialist economy when it was 'in every respect, economically, morally and intellectually, still stamped with the birthmarks of the old society' (Marx 1875,

[29] I have tried to pursue the latter question in my paper for the Bristol Conference on 'Practical Reason', Sen (1972).

[30] 'The Decision of the Central Committee of the Chinese Communist Party Concerning the Great Proletarian Cultural Revolution,' adopted on 8 August 1966, reproduced in Robinson (1969), p. 95. This is the so-called 'Sixteen Points'.

p. 21), the programme of 'cultural revolution' pleaded for 'an education to develop morally, intellectually and physically and to become labourers with socialist consciousness and culture'.[31]

The question of dissociating choices from individualistic preferences and individual welfare seems to have been fairly central to the Chinese experiment on work motivation and the cultural revolution.[32] The recurrent emphasis on acting 'without calculation of loss or gain' and the persistent attack on the pursuit of personal gains relate to this. It is a characteristic of the Prisoners' Dilemma type situation that the consequence of everyone acting rationally according to his true preferences and individual welfare is an inferior social outcome for all, and acting in a morally dogmatic way (*as if* one's preferences were different, whether or not they actually are so) can produce a superior outcome for all (even in terms of individual welfare functions, whether or not they take note of the welfare of others).

This type of consideration seems to have characterized an aspect of the cultural revolution and links it up not only with the Chinese experiments on payment methods in the Leap Forward period and later, but also with the mainstream of the socialist debate on the works principle *versus* the needs principle, involving diverse authors from Marx (1875) to Lerner (1944).

It is not my object here to assess the successes and failures of the Chinese experiment in trying to shift the emphasis of distribution policy from work to needs. What is important for our purpose is to place this experiment in the perspective of the chain of thought linking the Marxian analysis of socialist distribution on the one hand with the literature on optimal allocation and distribution on the other. This is of obvious relevance to the whole question of economic inequality in a socialist society, and the Chinese experiment crystallizes a significant aspect of it.

[31] 'The Sixteen Points', in Robinson (1969), p. 93.
[32] See Riskin (1971).

Desert and productivity

I should like to end the discussion with some remarks on the concept of desert itself. There are several alternative interpretations of desert that can be found in the economic literature. The marginal productivity theory has sometimes been viewed as a theory of deserts. This is explicit in the writings of some, e.g., J. B. Clark (1902), but its implicit presence can be felt in many other discussions of income distribution.[33]

In contrast, Marx's theory of exploitation provides an alternative theory of desert, giving labour the right to the whole of the net produce. The normative aspect of Marx's approach to the question has got somewhat overshadowed by debates on its descriptive features (e.g., the so-called 'transformation problem'), but there is no doubt that Marx saw his theory of value partly as a theory of desert.[34] This was not based on a denial that machinery can be productive—very much the contrary—but on the idea that labour in a direct *plus* 'embodied' form as 'the ultimate source of all value' deserves to enjoy the whole of the net output, and profits merely reflect a particular social arrangement of private ownership of means of production.[35]

The concept of 'exploitation' as developed by Joan Robinson (1933) took departures from the competitive value of the marginal product as indications of exploitation and two kinds were distinguished, viz., (i) 'monopolistic exploitation', given by the difference between the marginal revenue product and the competitive value of the marginal product (reflecting monopolistic elements in the product market), and (ii) 'monopsonistic exploitation', given by the difference between the wage rate and the marginal revenue product (reflecting monopsonistic elements in the labour market). The concept of desert here was a variant of the marginal productivity theory

[33] Paul Samuelson notes: 'To my astonishment I find that the arbitrariness of J. B. Clark's views on the deservingness of competitively determined rewards is not universally recognized' (Samuelson 1950a, p. 1577).
[34] See especially Part III of *Capital*, Volume I (Marx 1887).
[35] Marx did, however, treat 'nature' as an ultimate source of value as well (see Marx 1875, p. 17).

and was presented within that framework of thought, which she did, of course, subsequently reject.[36]

Sometimes desert has been viewed in terms of the appropriate prices p 'associated' with an optimal programme. These are prices that would 'sustain' that programme in the sense that the people involved would on their own make the choices appropriate for that optimum if they did their gains-maximizing calculations at those prices.[37] Such 'associated' prices need not always exist even when an optimum exists with respect to the objective function and the constraints, and much depends on the nature of the economic assumptions made (e.g., whether there are increasing returns to scale, external economies, etc.).

A special case of such an optimization exercise is that of achieving Pareto optimality. Given certain assumptions, any set of prices emerging in a competitive equilibrium would do for this purpose.[38] Since in the neo-classical framework the competitive price of factors of production would equal the respective marginal productivities, this could provide another approach to viewing marginal productivity as an interpretation of desert. However, since Pareto optimality is a very limited objective (see Chapter 1), the normative appeal of this approach may not be particularly great even within the neo-classical framework.[39]

Productivity and ability

A more full-blooded concept of desert than prices 'associated' with an optimum is based on the notion of 'ability'. Two distinctions between this idea and that of productivity must be noted. First, the productivity idea relates to all factors of production while the notion of ability relates essentially to labour. There are 'fertile' pieces of land but not 'able'

[36] Robinson (1956), (1960).

[37] See Dorfman, Samuelson and Solow (1958), Arrow and Hurwicz (1960), and Malinvaud (1967).

[38] See Debreu (1959) and Arrow and Hahn (1972). For an illuminating informal presentation, see Koopmans (1957).

[39] See Meade (1965).

pieces, nor do we run into 'able' machines. Thus the framework of ability does not directly apply to the question of property incomes. Second, even within labour, productivity can be distinguished from ability as such, since (i) opportunities for the use of one's abilities may not arise in a particular situation, and (ii) 'innate abilities' may be distinguished from derived competence reflecting education, training, and opportunities of learning.

It is this last distinction that has come much into focus in the context of the recent emphasis on 'equality of opportunities', which is in fact a desert-based concept. While educational expansion in modern Western societies has often been put forward as evidence of growing equality of opportunities, serious doubts about the achievements in this field have been raised in a number of studies.[40] It is not my intention here to go into the empirical correctness of the thesis, but to see this approach as falling within the corpus of desert-based normative theories.

A distinction between a system of rewards according to ability and that related to 'associated prices' with an optimum programme is also worth noting here. Natural talents are one thing to which the question of incentives is irrelevant, since people cannot set their natural talents aside in response to a price cut. Given an inflexible supply of talents, there will not be a *unique* 'optimal' price associated with it, since the same supply of talents will obtain at different rates of reward.[41]

It is difficult to justify rewarding talents on grounds of efficiency. We find here two alternative concepts of desert locked in combat with each other. One demands—on grounds of 'merit'—a higher reward for natural ability and does not

[40] See, in particular, O.E.C.D. (1971) and Bowles (1972). See also Klappholz (1972).

[41] In the short run the inflexibility assumption is clearly appropriate. In the long run variations of the population size could be relevant, and it might be argued that there would be incentive effects if (i) a lower reward to talents were to reduce the propensity of the talented to procreate, and if (ii) talented parents had a greater than average probability of giving birth to talented children. While (ii) seems to be under much discussion today, the argument holds only if (i) is also correct, and there is, in fact, very little evidence for it.

accept the claims of acquired competence which reflects social arrangements. The other points towards rewarding acquired abilities—on grounds of 'incentives'—but provides no case for rewarding natural talents. Both, of course, conflict with the notion of needs.

Desert and needs

In this book my emphasis has been primarily on needs, and the analytical framework presented here is biassed in that direction. There are a number of reasons for this. First, as we have just now seen, there are alternative interpretations of the concept of desert and they can conflict sharply. There seems to be more unity in interpreting the concept of needs.

Second—and here I reveal my bias—it seems to me arguable that needs should have priority over desert as a basis for 'distributional' judgements as such, to which the concept of 'inequality' belongs. Of course, as argued earlier, inequality evaluation involves *non-compulsive judgements*, but *within that sphere* none of the conceptions of desert seem more appropriate.

(1) Taking up first the *incentive-oriented* interpretation of desert, a system of incentives would appear to be a means to an end rather than an end in itself, whereas the fulfilment of needs would be usually taken to be a good thing in itself. If an incentive-oriented unequal distribution—unrelated to needs —is defended, it seems reasonable to describe it as something defended on 'non-distributional' grounds, e.g., the total size of income. If, on the other hand, relative needs are manifestly different and an unequal distribution corresponding to differences in identified needs is recommended, the defence of this position would seem to be on 'distributional' grounds themselves.

(2) Coming now to the *merit oriented* system of desert, giving more income to the naturally talented people does, of course, amount to giving less to those without talents. The latter includes the Thalidomide babies of today who will be adults tomorrow, the old and the infirm stripped of their talents by the natural process of aging, and—of course—the genetically defective. A system based on needs would seem to have greater use for the complex idea that we call humanity. Even for limited application of the merit principle—giving more than the 'norm' to the specially meritorious but not less than the 'norm' to the demented—it can be argued that the measure of merit is culture-specific. While many of us may be content to live in a society which values the ability to lecture more than it values, say, the ability to make loud, shrill noises by blowing sharply through one's nose, we might be perfectly able to give long lectures about possible societies in which the latter quality would be the more desired virtue. Merit is a bit of an accident not only in its origin, but also in its being treated as merit.

(3) The Marxian principle of desert based on the value of labour has been a powerful mover of mankind in providing a focus of attention on inequalities arising from class differences in the ownership of means of production, but—as we saw—Marx himself regarded this right to the 'fruits' of labour as a 'bourgeois right' to be supplanted by the principle of needs when the opportunity arose. As a critique of property incoma, this notion of labour 'getting its value' has an obvious appeal, but it is difficult to defend it as a 'principle' against that of distribution according to needs, if feasible. And the question of feasibility takes us back to incentives, cultural values, and the question of tolerating inequality on 'non-distributional' grounds; these questions have been discussed earlier in this chapter.

(4) It is not easy to interpret the neo-classical marginal productivity theory as a normative theory, as was pointed out earlier, and if it does have a place it is a part of an incentive system corresponding to prices associated with an optimal programme. But even in the neo-classical model the only optimality such 'competitive prices' guarantee is merely Pareto optimality, which is in itself a very limited goal. Furthermore, as shown earlier, the presence of 'rent' elements in the high payments to the talented, productive people also makes the incentive problem less straightforward.

It is with this general outlook that I have concentrated in this work on analysing the evaluation of inequality mainly from the point of view of needs rather than that of desert. While relatively little help could be obtained from the main avenues of welfare economics—'old' and 'new'—we have used a broad framework of interpersonal comparisons (formalized in $\tilde{R}$) and have analysed principles of evaluation and statistical measures of inequality in that light. Because of the mixture of descriptive and normative considerations in the concept of inequality and the inherent incompleteness of that concept, inequality evaluation has been seen in terms of non-compulsive, evaluative judgements expressed as quasi-orderings. The alternative approaches explored would all fall within this general framework.

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