CHAPTER 1

The limits to choice

Consumer behavior is frequently presented in terms of preferences, on the one hand, and possibilities on the other. The emphasis in the discussion is commonly placed on preferences, on the axioms of choice, on utility functions and their properties. The specification of which choices are actually available is given a secondary place and, frequently, only very simple possibilities are considered. In this book, we shall have a great deal to say about preferences, and discussion of them begins in Chapter 2. We begin, however, with the limits to choice rather than with the choices themselves. Unlike preferences, the opportunities for choice are often directly observable so that, to the extent that variations in behavior can be traced to variations in opportunities, we have a straightforward and objective explanation of observed phenomena. It is our view that much can be so explained and that the part played by preferences in determining behavior tends to be overestimated. Hence, this first chapter considers what can be said about behavior without detailed consideration of how choices are made. A large part of this book, from Chapters 2 to 9, works with one very special assumption about the opportunity set, namely that choices are constrained by fixed, known prices in such a way that the total value of the objects chosen should not exceed some predetermined total. In this case, we say that the consumer faces a linear budget constraint. A detailed examination of more complex situations is postponed until Chapters 10 to 14 by which stage the basic material will have been covered. Although this later analysis is technically more difficult, nonlinear budget constraints arise frequently in practice and in §1.1 we present a largely diagrammatic survey of both the linear and nonlinear cases. This provides an elementary introduction to the later material as well as providing a preview of the topics to be covered in the rest of the book. For uniformity of usage, the limits to choice will be described through the “constraints” facing consumers even though, for the moment, nothing is being optimized. Indeed, no formal assumptions are made about choice itself; for the most part, the likely implications of the constraints under consideration will be obvious enough.

Section 1.2 focuses on the simple linear budget constraint that underlies much of the subsequent analysis. Again, without specific assumptions about choices, we can make quite far-reaching deductions about behavior and we
explore the consequences of these for empirical analysis. Finally, as a preparation for Chapter 2, we use a model of *irrational* choice proposed by Becker (1962) to suggest that the use of preference orderings is only one way in which models of consumer behavior may be completed.

### 1.1 The nature of opportunity sets

**The linear budget constraint**

The simplest and single most important type of opportunity set is that which arises when the household has an exogenous budget, outlay or total expenditure $x$, which is to be spent within a given period on some or all of $n$ commodities. These can be bought in nonnegative quantities $q_i$ at given fixed prices $p_i$. The constraint can then be written

$$ x \geq \sum_{i=1}^{n} p_i q_i \quad (1.1) $$

When $n = 2$, (1.1) is illustrated in Figure 1.1 by the area in the triangle $AOB$ since $q_1$ and $q_2$ must be nonnegative. The coordinates of the points $A$ and $B$ are marked; at $A$ all of the budget is spent on good 2 so that $q_2 = x/p_2$ and $q_1 = 0$ and conversely at $B$. Note an immediate complication which arises if there is a basic survival constraint. An example is illustrated in the diagram.

![Diagram](image)

**Figure 1.1. A linear budget constraint with a survival constraint.**

### 1.1 Nature of opportunity sets

Here $q_1^{\min}$ and $q_2^{\min}$ represent the minimum quantities of the two goods necessary for survival within the period. Hence, choice is restricted to the triangle $DCE$, so that for a household with a budget as low as $x = p_1 q_1^{\min} + p_2 q_2^{\min}$, there is no choice; it must buy at $C$ or cease to exist. In general, the survival constraints may take a variety of forms; for example, with very little shelter, more food is needed for survival than with a great deal of shelter.

The formalization of choice within a linear budget constraint is the main topic of Chapter 2. In spite of its limitations, of which more below, reinterpretation of the $p_i$'s, $q_i$'s, and $x$ allows a wide field of application for (1.1), given suitable, but often restrictive assumptions. If $q_i$ is taken to be leisure and $q_j$ some composite of goods, then Figure 1.1 can be interpreted as the situation facing a consumer with a fixed money wage, no income tax (other than a proportional one) and no unearned income. We shall follow this further in §4.1. Alternatively, $q_1$ and $q_2$ may be taken as composites of goods in two different periods with $p_1$ and $p_2$ as the two corresponding price levels. If the consumer begins period one with no assets, if he can save or borrow at an interest rate of zero, and if income arising in the two periods, $y_1$ and $y_2$, is to be spent, the budget constraint is

$$ y_1 + y_2 \geq p_1 q_1 + p_2 q_2 \quad (1.2) $$

which, with $x$ equivalent to $y_1 + y_2$ is simply equation (1.1). As we shall see in §4.2, assets and interest rates can be incorporated into (1.2) without losing the analogy with the basic constraint (1.1). The analogy can even be extended to purchases of durable goods, albeit under restrictive assumptions, again see §4.2. Finally, consider the more radical reinterpretation of (1.1), which takes $x$ not as individual expenditure but as national income or expenditure and which interprets the $q_i$'s as the incomes of each of the individual households. In this analogy, taking each of the $p_i$'s as unity makes (1.1) the problem faced by the government in allocating a fixed total income between its citizens. Further analysis of this problem is in Chapter 9, especially §9.2.

**Nonlinear constraints**

Implicit in the linear budget constraint is the institutional setting of efficient markets with negligible transactions costs. Therefore, as our first example of nonlinear constraints, it is instructive to take the example of a barter economy. Let $A$ in Figure 1.9 be a household's initial endowment of food and clothing. Without a general medium of exchange, information and transactions costs prevent the household which wishes to trade clothing for food along $AB$ from being in touch with those willing to trade food for clothing along $AC$ so that the rate of exchange differs in the two directions. It is also likely that different households will face different divergent pairs of rates of
exchange. The development of trading posts with some centralization of
transactions is likely to reduce the divergence between the rates of exchange
while, it can be argued, a fully monetized competitive economy will effectively
eliminate it altogether. However, we shall later see examples where, even in a
developed market economy, informational problems are likely to cause diver-
gences to occur between buying and selling prices faced by a given house-
hold.

Even in the simplest case of spending a fixed total on specified goods within
a single period, nonlinearities may arise. In many countries, for example,
households face a two-part or even three-part tariff in paying for fuel such as
gas or electricity. The first few units typically come at a high price with extra
units at a reduced rate giving rise to the budget constraint illustrated in Fig-
ure 1.3. A similar situation occurs if discounts are given on bulk purchases.
An extreme case of nonlinear pricing that works in the opposite direction is
when the price becomes infinite above some specified level of purchases. This
is precisely equivalent to the imposition of a ration in the form of an upper
limit on purchases, as occurs, for example, in wartime. Rationing of a dif-
ferent type may occur when a household has no direct control over its con-
sumption of some particular good such as housing, in the short run, or
defense expenditure. In this case, the household may be constrained to con-
sume more than it wishes. We return to the topic of rationing in §4.3

Nonlinearities become even more pervasive when we move to questions of
leisure choice or intertemporal behavior. Take first the case where the con-
sumer can choose the number of hours he can work, faces a fixed wage rate,
ω, but has some transfer income, μ, say. Hence if T is the total number of
hours available and qₙ is leisure, the budget constraint is

\[ μ + ω(T - qₙ) \geq \sum p_i q_i \]  

(1.3)

Figure 1.4 illustrates this for two values of ω, ω¹ and ω². Note that this budget
constraint, unlike that of Figure 1.1, is kinked outwards at A. Kinks of this
type are characteristic of many of the nonlinear budget constraints we shall
consider. Points such as A are of considerable practical importance since there

turn out to be good theoretical reasons for expecting a fraction of consumers
to make that particular choice. We shall take up this point formally later; in
the present example, the choice between A and some point between A and B¹
or B² is the choice whether or not to participate in the labor force and this
question will be discussed in §11.1. In general, we can think of consumers as
being continuously distributed along the budget line according to tastes, cir-
cumstances, and so on, and if points to the right of A were feasible, we should
expect some consumers to choose them. As it is, all such consumers must
make do with A so that, in contrast to any other point along AB, a finite pro-
portion of the population will be found at A itself.

In practice, Figure 1.4 is likely to be too simple. In Figure 1.5, beyond a cer-
tain number of hours of work (to the left of C), hours are paid at a higher
overtime rate, but if hours go to the left of D, the worker becomes liable for
income tax. We thus have two outward kinks (at D and B) and an inward kink
(at C). When we come to consider specific models of how choices are made, we
shall see that such kinks have important consequences since, under many such models, points with outward kinks are rather frequently chosen with the opposite being true for inward kinks. As a final example in the leisure-goods context, note that budget sets may not be areas as illustrated so far but rather points or lines. Figure 1.6 illustrates the available choices when there are three jobs each with different hours of work and wage rates. Clearly, there is a wide range of possibilities that combine elements of Figures 1.4 to 1.6 and some of these will be discussed in Chapter 11. In that chapter, we shall argue the importance of correctly specifying choice sets for the understanding and interpretation of labor market behavior. In the broader context, the analysis of voluntary and involuntary unemployment and hence of much of macroeconomics hinges on the constraints which consumers are assumed to face in the labor market.

Different, but equally important nonlinearities arise when we turn to opportunity sets in which goods or leisure are distinguished according to the periods in which they are consumed. Equation (1.2) illustrated the linear budget constraint for two periods with a zero interest rate and no assets. If the consumer in this model wishes to spend more than his income in period 1, borrowing with repayment in period 2 will be required. However, consumers are not always able to borrow so that, as illustrated in Figure 1.7, the budget constraint is not ABC but rather ABD so that, once again, we have an outward kink. Consumers at B spend all their current income and, for as long as they remain there, have a marginal propensity to consume of unity. If we make
this situation less extreme by allowing for interest rates, but assume that the consumer pays a higher rate for borrowing than for lending, the budget constraint will still be kinked but less severely. The line $ABE$ in Figure 1.7 illustrates a higher rate for borrowing than lending. Situations similar to these are discussed in Chapter 12, while in Chapter 13, on durable goods, additional nonlinearities are considered that arise through imperfect secondhand markets (where buying and selling prices differ) and the discreteness that is due to the indivisibility of many durable goods. As in the leisure case, the distinction between unfettered choice along a linear intertemporal budget constraint and relatively constrained choice at kinks like $B$ in Figure 1.7 turns out to be crucial for macroeconomic analysis. Indeed, the existence and properties of the Keynesian consumption function cannot be understood without the existence of nonlinearities in both leisure and intertemporal budget constraints.

One of the important postwar developments in consumer theory has been in the construction of models in which the household is viewed as a producer, combining market goods with one another and with leisure through household production functions. The object of this activity is the production of a limited number of basic goods that are regarded as the real object of consumer choice. If there are two such basic goods $z_1$ and $z_2$, their output is limited by the time available for household production and by the wage rate and the prices of market commodities. If the production functions allow smooth substitution between time and market goods as inputs, a typical budget constraint might be as illustrated in Figure 1.8; the analogy with the cost-minimizing firm is clear. Frequently, the technology is assumed to be of the fixed coefficient type. Figure 1.9 illustrates the famous "diet" problem in which a household requires two outputs $z_1$ (protein) and $z_2$ (calories). These are provided by four different market foods which, if total food expenditure is spent on them, lead to points $A$, $B$, $C$, and $D$. Points along the line segments can be reached by buying mixed bundles, but the constraint stops short of the

Figure 1.8. The budget constraint for the outputs of household production.

Figure 1.9. The budget constraint for the diet problem with four market goods.
axes because there may well be no foods which are 100 percent protein or 100 percent calories. We discuss household production models further in Chapter 10.

As a final example that goes beyond individual choice, consider again the case of a government attempting to allocate total output amongst its citizens. If total output is independent of its allocation, the constraint is the linear one at 45 degrees to the axes. In practice, however, a guaranteed equal allocation is likely to affect the incentives to work and to produce so that egalitarian distributions may well be associated with low total output and vice versa. An extreme case is illustrated in Figure 1.10, where $x_1$ and $x_2$ are the allocations to each of two households. Attempts to ensure an egalitarian allocation in the region of A will ensure that both households are relatively badly off. Alternative solutions, however, can only produce a large allocation for one household and not for both. Optimal solutions to such problems are complex and we discuss the issues involved in Chapter 9.

**Information and consumer perceptions**

To understand consumer behavior, we must recognize that the budget constraint that is relevant is the constraint perceived by the decision maker. In many of the examples already given, full information on the opportunities available may not be easily available to everyone. In the leisure-goods choice, for example, many countries have extremely complicated tax and social security systems and it is clear that many consumers are not perfectly informed. Again, in Figure 1.10, the government is in practice unlikely to have anything but the vaguest information on the possibilities of trading off efficiency for equity in the population at large. However, the invalidity of the perfect information assumption is nowhere more serious than in models of choice over time. The future is inherently uncertain. The consumer-worker may become unexpectedly unemployed, his house may burn down or his financial assets may be eroded by unexpectedly high rates of inflation. A formal analysis of uncertainty is rather difficult and in some ways rather unsatisfactory (see Chapter 14). But, at least under some models, uncertainty in the intertemporal context will add a precautionary element to behavior, particularly as regards saving and the holding of liquid assets. Such behavior is strongly influenced by the constraint facing consumers and, once again, kinks in budget sets are important. In Figure 1.7, the consumer who cannot borrow (ABD) is in a much worse position to withstand unexpected financial misfortune than is the consumer who can (ABE or ABC). Similarly, an individual who knows that durable goods bought this period can only be sold next period at a heavy discount will be cautious in his present purchases and will be particularly keen to avoid buying too much. Chapters 12, 13, and 14 take up these issues in depth.

**Interactions between agents**

For many problems, it is adequate to model consumers individually, each making his own choices subject to his own budget constraint. Group behavior can then be derived as the sum of its parts. Sometimes, however, the opportunities available to any one consumer depend crucially on what others do, and opportunity sets must then be modeled to reflect this dependence. A classic example is the public-goods provision where the impossibility of excluding any individual from the enjoyment of the good once it has been provided renders an individualistic analysis inappropriate. The amount of national defense available to me depends, not on what I am prepared to finance (or only infinitesimally), but on what is already being provided by my fellow citizens. A consumer who takes this into account will always show an unwillingness to buy more of the public good, since he gets such poor value for his money, yet this does not reflect the true worth he attaches to extra units of provision. Recent, highly ingenious work has gone into the problem of how to redesign budget sets (via taxes and subsidies) so that consumers will choose on an individualistic basis those quantities that fulfill their collective needs.

One of the dangers of not recognizing interdependencies between consumers is that we can mistake highly constrained behavior for free choice. If we wish to attach welfare significance to behavior, such a mistake is of the highest importance, since freely chosen positions (end to be regarded as optimal while those adopted under coercion are clearly not so. A classic example of this at the individual level is the assurance game, see Sen (1967), typical data for which are given in Table 1.1. Prisoners A and B are both guilty and are being separately questioned. If both maintain their innocence, both are
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<table>
<thead>
<tr>
<th>Action of A</th>
<th>Action of B</th>
</tr>
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<tbody>
<tr>
<td>Confess</td>
<td>Not Confess</td>
</tr>
<tr>
<td>5(A), 5(B)</td>
<td>2(A), 20(B)</td>
</tr>
<tr>
<td>Not confess</td>
<td>20(A), 2(B)</td>
</tr>
<tr>
<td>0(A), 0(B)</td>
<td></td>
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</tbody>
</table>

discharged and this is clearly the preferred position for both. However, if A maintains his innocence but B confesses, A will be jailed for twenty years so that the "not confess" strategy is an extremely risky one. By contrast, confession, although ruling out an absolute discharge, can lead at the worst to five years' imprisonment. Hence, if A has little taste for risk for its own sake, he is likely to choose the safer strategy, especially if he knows that B is making similar calculations and that B knows that he knows, and so on. If these arguments are correct, both prisoners are likely to confess although this outcome makes them both worse off than if they had both refused to do so.

Such examples are not merely philosophical paradoxes. What Keynes called the "fallacy of composition," that the whole is not merely the sum of its parts, is an essentially similar phenomenon. If employers knew that the unemployed workers they do not employ would spend the extra income they do not earn on the goods the employers do not produce because of insufficient demand, collective action would reduce unemployment although individual action cannot. If such constraints are not recognized, we are forced into the patent absurdity of interpreting mass unemployment as voluntarily chosen leisure and hence condemning any policy that attempts to remedy it.

1.2 The implications of a linear budget constraint

Notwithstanding the importance of nonlinearities and of the other issues in §1.1, a great deal of consumer demand analysis is built on the assumption of a simple linear budget constraint. We write this in equality form

$$x = \sum_k p_k q_k$$

(2.1)

with total expenditure $x$, prices $p_k$ and quantities $q_k$. The use of the equality, as opposed to the inequality of (1.1), will be justified if consumers always attain the upper boundary of the opportunity set. This will happen if the consumer cannot completely satisfy all his wants within the budget constraint; there is always some good more of which is desirable. The use of (2.1) rules out the nonlinearities, indivisibilities, uncertainties, and interdependencies of

1.2 Implications of a linear budget constraint

§1.1 It also assumes that the total amount to be spent $x$ is decided separately from how its details are to be made up. Possible justifications for this will be discussed in Chapter 5. Note carefully that $x$ is total expenditure and not income; we shall maintain a close distinction between these concepts throughout the book.

Properties of demand functions

Assume that demand functions exist; the consumer, by some means or other, has rules for deciding how much of each good to purchase faced with given prices and total outlay. We write these functions as

$$q_i = g_i(x, p)$$

(2.2)

These relationships giving quantities as a function of prices and total expenditures we shall refer to as Marshallian demand functions; other types of demand functions will be introduced in Chapter 2. We shall assume that the Marshallian demand functions are continuously differentiable and postpone also to Chapter 2 any consideration of why this might be so.

The fact that the demand functions satisfy the budget constraint (2.1) immediately places a constraint on the functions $g_i$. Specifically,

$$\sum_k p_k g_k(x, p) = x$$

(2.3)

and as we shall see below, this can by no means be satisfied by any arbitrary selection of functions $g_i$. Equation (2.3), for obvious reasons, is referred to as the adding-up restriction. But this does not exhaust the implications of the budget constraint. Consider some vector of purchases $q$, and assume that it satisfies (2.1) for prices $p$ and outlay $x$. Then, since the constraint is linear and homogeneous in $x$ and $p$, the vector $q$ will also satisfy the constraints for any multiple of $x$ and $p$. If total expenditure and prices are twice as high, the constraint is the same. Formally, for any positive number $\theta$ and, for all $i$ from 1 to $n$,

$$g_i(\theta x, \theta p) = \theta g_i(x, p)$$

(2.4)

This restriction implies that the demand functions are homogeneous of degree zero, so that (2.4) is known as the homogeneity restriction. It is also known as "absence of money illusion" since the units in which prices and outlay are expressed have no effect on purchases. Note that to get this result we have made a weak but not trivial assumption about behavior. This is that prices and outlay play no role in choice other than in determining the budget constraint, so that the units in which prices and outlay are measured have no effect on the consumer's perception of opportunities. One case that violates this as-
Extensions to the basic model

The model of consumer behavior developed in Chapter 2 and applied in Chapter 3 is clearly limited both in scope and in realism. The process of extending and improving the model in various directions will occupy much of the rest of the book and is the particular subject matter of Part Four. In this chapter, we make a start on these extensions but only in a very limited way. In particular, in §4.1 and §4.2, we show how labor supply decisions, savings behavior, and purchases of durable goods can be handled within the framework of utility maximization subject to a linear budget constraint, provided the arguments of the utility function and the constraint are appropriately redefined. We shall refer to these models as neoclassical, a name we use to label the assumption of linear budget constraints with fixed, known prices. In the present context, none of the neoclassical models turns out to be very realistic and hence the substantial attention paid to their improvement in Part Four. Nevertheless, it is extremely important to understand them. First, they play an important part in much contemporary economic analysis, and it is essential to understand exactly the assumptions on which their theoretical validity depends. Second, they yield important insights that we ignore at our peril and, in Part Four, they will be the platform on which we attempt to build more realistic and relevant models.

Section 4.3 is an exposition of the theory of demand when the amounts of some of the commodities are fixed. This important extension of the model is relevant for the discussion of short-run versus long-run demand, for the analysis of rationing and of public goods, and for the understanding of much of the modern macroeconomic literature that treats consumers as being sometimes constrained to given quantities in either the goods or labor markets. We also discuss how shadow prices can be derived and how they fit into the apparatus so far developed.

4.1 The simple neoclassical model of labor supply

Our main analysis of labor supply is in Chapter 11; the following elementary neoclassical description is an essential prerequisite for the later work.

The budget constraint and preferences

We start our analysis by applying the theory of choice to the simplest neoclassical specification of constraints. Here we have an individual or a household containing a single worker faced with the choice of buying different bundles of goods at a given price vector, with these purchases being financed out of a given nonlabor income $\mu$ and labor income $\omega \ell$, where $\omega$ is the given wage rate and $\ell$ is the amount of time the individual chooses to work. Formally,

$$\mu + \omega \ell = \sum p_i q_i$$

(1.1)

since the nonsatiation axiom guarantees an equality rather than an inequality.

In addition,

$$q_i \geq 0, \quad \text{all } i \text{ and } T \geq \ell \geq 0$$

(1.2)

meaning that negative amounts of goods cannot be bought, negative amounts of work time cannot by supplied, and labor supply cannot exceed the time endowment $T$. If flows were measured daily, we could think of $T$ as 24 hours minus the time necessary for sleeping and other minimal maintenance tasks. But we could also measure the flows over longer time periods such as a year or more. In some ways, a long-run interpretation would justify the neglect of saving and would overcome the criticism that in the short run, most workers have to work the hours required by the job specification: in the long run, we can argue that workers can choose their hours by choosing between different jobs. On the other hand, it then seems implausible to maintain the linearity of the budget constraint since the wage rate is unlikely not to vary with the number of hours supplied. However, let us adopt it as a simplifying approximation.

Let the utility function be

$$u = u(q_0, q_1, \ldots, q_n)$$

(1.3)

where we denote leisure $T - \ell$ by $q_0$ to make clear that leisure is comparable to other goods with the wage rate as its price. For simplicity of presentation, we shall assume in much of what follows that there is only a single consumption good and give brief indications of how the results generalize when there are $n$ such goods. Working with one consumption good is not quite as unrealistic as it appears: if the $n$ relative prices are constant, we shall see in Chapter 5 that the price weighted total of the consumption vector (the "Hicks aggregate") can be treated like a single good. Since in cross sections we assume that everyone faces the same relative prices for goods, this one good model is the natural one to adopt for cross-section analysis. We can picture the problem of maximizing (1.3) subject to (1.1) and (1.2) in Figure 4.1. The attainable combinations of consumption and leisure are given by the quadri-
4 Extensions to the basic model

4.1 Simple neoclassical model of labor supply

and above those of a change in the price of a good and this crucially alters the analysis of income and substitution effects. The quantity $\mu + \omega T$ represents the total purchasing power available to the consumer to be spent on leisure and goods and is usually called full income. We shall denote it by $X$ since, not including saving, it is a concept more akin to total expenditure as used up to now.

Before going on, consider the effects of taxation on the budget constraint (1.4). In dealing with commodity prices, it is not necessary to separate the tax component from the producer price, but with labor supply, the situation can be more complicated since, even when only linear taxes are considered, the tax affects both labor and nonlabor income. In this case, however, $\omega$ and $\mu$ can be reinterpreted as net concepts. Suppose, for example, that the same tax rate $\tau$ applies to labor and to nonlabor income; hence,

$$\text{tax paid} = \tau(\mu + \omega T) - b$$

(1.5)

where someone with no income receives a benefit of $b$. This is a simple “negative income tax” system in that those below a certain threshold receive net benefits in proportion to the shortfall of their income just as those above that level pay taxes in proportion to the excess of their income above the threshold. After-tax income is $(1 - \tau)(\mu + \omega T) + b$ so the budget constraint is now

$$pq + (1 - \tau)\omega q_0 = b + (1 - \tau)\mu + (1 - \tau)\omega T$$

(1.6)

or

$$pq + p_0 q_0 = \mu^* + p_0 T$$

(1.7)

where $p_0$ is the net wage $(1 - \tau)\omega$ or price of leisure and $\mu^*$ is net nonlabor income $b + (1 - \tau)\mu$. Otherwise, (1.7) is identical to (1.4). Changes in the tax rate $\tau$ change the net wage and net nonlabor income so that the income effect of tax changes has an extra component compared with a change in the wage. In practice, tax systems are often not linear, and we shall discuss some of the problems this causes in Chapter 11.

Income and substitution effects and the Slutsky equation

Figure 4.2 compares the income and substitution effects of a change in the wage with the more usual ones of a change in a commodity price. If $B$ is the starting point on the budget line $AC$, an increase in $\omega$ will cause an upward rotation around $A$ to $AF$. Contrast this with the traditional case when an increase in price of the good on the horizontal axis causes the budget line to rotate inwards, from $AC$ to $CG$. The difference between $CG$ and $FA$ is because of the extra income effect of the change in $\omega$ on full income $X$ through the revaluation of $T$. Thus, although the substitution effect of the price change is, as usual, the move from $B$ to $E$, the total income effect is the move, not from $E$ to $B$, but
Extensions to the basic model

4.1 Simple neoclassical model of labor supply

It is easy to see that when \( p \) and \( q \) are vectors, (1.11) becomes \( q_i = h_i(u, \omega, p) = \partial c(u, \omega, p) / \partial \omega_i \) for \( i = 1, \ldots, n \). As in Chapter 2, the solution to the problem “maximize utility (1.3) subject to (1.4)” is given by the Marshallian demand functions

\[
q_\omega = g\omega(u + \omega T, \omega, p) = g\omega(X, \omega, p) \tag{1.12}
\]

\[
q = g(u + \omega T, \omega, p) = g(X, \omega, p) \tag{1.13}
\]

with the appropriate vector extensions. Labor supply equations are given by (1.10) and (1.12), using \( \ell = T - q_\omega \). As in the commodity case, substitution of (1.9) for \( u \) in the Hicksian demand leads to the Marshallian demands, while writing \( u + \omega T = X = c(u, \omega, p) \) and substituting in the Marshallian demands leads back to the Hicksian demands.

The Slutsky equations may now be derived straightforwardly. Paralleling the discussion of Figure 4:2, we wish to decompose the total effect on the leisure of a wage change into a substitution effect and an income effect. The total effect of \( \omega \) on \( q_\omega \) is given, via (1.12) as

\[
\frac{\partial q_\omega}{\partial \omega} = \frac{\partial g\omega}{\partial \omega} + \frac{\partial g\omega}{\partial X} . T + \frac{\partial c}{\partial \omega} \times \frac{\partial \omega}{\partial X} \frac{\partial X}{\partial \omega} \frac{\partial \omega}{\partial \omega} \tag{1.14}
\]

Diagrammatically this is the decomposition of the move from \( B \) to \( B_1 \) into moves from \( B \) to \( B_2 \) and from \( B_2 \) to \( B_1 \). We shall now show that the first term, the wage effect holding constant, that is, without adjusting full income upwards for the revaluation of the time endowment, decomposes into a substitution effect and an income effect in the usual way.

From (1.12) and the cost function

\[
q_\omega = g\omega \left[ c(u, \omega, p), \omega, p \right] = h_\omega(u, \omega, p) \tag{1.15}
\]

Differentiating with respect to \( \omega \) gives the substitution effect as

\[
\frac{\partial g\omega}{\partial \omega} = \frac{\partial g\omega}{\partial \omega} + \frac{\partial g\omega}{\partial X} \frac{\partial c}{\partial \omega} + \frac{\partial g\omega}{\partial \omega} \frac{\partial \omega}{\partial X} \frac{\partial X}{\partial \omega} \frac{\partial \omega}{\partial \omega} \tag{1.16}
\]

The first term, which since \( \partial c/\partial \omega = q_\omega \) equals \((\partial q_\omega/\partial \omega)q_\omega\) is the conventional income effect and diagrammatically is the move from \( B_2 \) to \( E \). If leisure is a normal good, it is positive. Substituting (1.16) into (1.14) gives

\[
\frac{\partial q_\omega}{\partial \omega} = \frac{\partial g\omega}{\partial \omega} + \frac{\partial g\omega}{\partial X} \frac{\partial c}{\partial \omega} + \frac{\partial g\omega}{\partial \omega} \frac{\partial \omega}{\partial X} \frac{\partial X}{\partial \omega} \frac{\partial \omega}{\partial \omega} \frac{\partial \omega}{\partial \omega} \tag{1.17}
\]

Diagrammatically, the first equality in (1.17) decomposes the move \( B \) to \( B_1 \) (total effect) into \( B \) to \( B_2 \) (substitution effect), \( B_2 \) to \( E \) (conventional income effect, negative if leisure is normal) and \( B_2 \) to \( B_1 \) (revaluation-of-time-endowment effect, always positive). Note that the total income effect cannot be negative.
Extensions to the basic model

This Slutsky equation can also be written in terms of labor supply as

\[
\frac{\partial c}{\partial \omega} = \frac{\partial c}{\partial \mu} + \frac{\partial c}{\partial X} \cdot \ell
\]  

(1.18)

which looks (rather misleadingly) like a normal Slutsky equation. Note carefully, however, that the concavity of the cost function implies that \( \partial q / \partial \omega \) (= \( \partial \ell / \partial \omega \)) is nonpositive so that \( \partial c / \partial \mu \) is nonnegative; similarly, \( \partial c / \partial X \) is normally negative rather than positive. Hence, it is perfectly normal for the labor supply curve to be downward sloping (\( \partial c / \partial \omega < 0 \)) for at least some levels of \( \omega \) and \( \mu \). One possibility is illustrated by BB' in Figure 4.3, where labor supply is measured right to left from D on the horizontal axis. Whether labor supply curves are backward bending in practice and if so, for which kind of workers, is a major empirical question and of obvious practical relevance, for example, for analyzing the incentive effects of income taxation.

Finally, we consider the effect of a change in \( p \), the price of goods, on the supply of labor. Since full income is independent of \( p \), differentiation of (1.15) and rearrangement gives a conventional Slutsky equation

\[
\frac{\partial q}{\partial p} = \frac{\partial q}{\partial \ell} - \frac{\partial q}{\partial X} \cdot q
\]  

(1.19)

or, in terms of labor supply

\[
\frac{\partial c}{\partial p} = \frac{\partial c}{\partial \ell} - \frac{\partial c}{\partial X} \cdot q
\]  

(1.20)

4.1 Simple neoclassical model of labor supply

(Note that in all these formulas, we could perfectly well replace derivatives with respect to \( X \) with derivatives with respect to \( \mu \).) If instead of one consumption good we may have many, (1.19) and (1.20) still hold with \( p \) replaced by \( p_i \) and \( q \) by \( q_i \). Although consumption goods as a whole cannot be complementary to leisure, individual goods can be. If the \( r \)th good is complementary to leisure, the compensated demand for leisure falls as the price of the good goes up so that compensated labor supply increases. Note finally that we could generalize the approach of this section to the case where the household has initial endowments not only of time but also of some or all of the goods. This would give the individual "excess demand functions" to be found in treatments of the theory of competitive general equilibrium—see Arrow and Hahn (1971), for example. In Chapters 12 and 13, we shall see that there are two classes of goods, financial assets and consumer durables, where the value of initial endowments is important for deriving sensible econometric specifications of demand functions.

Extensions and the specification of demand functions

The simple model of the previous subsections can be extended in a number of useful ways, and we shall consider two possibilities here, both of which are consistent with the preservation of a linear budget constraint. More complex extensions are reserved for Chapter 11. Consider first the case in which there are several potential earners in a household. The family utility function is then

\[
u(q_1, q_2, q, q_3) \]  

(1.21)

where \( q_1 \) and \( q_2 \) are the leisure times of the two individuals. In any cross-sectional application, this utility function would also contain a vector of given household characteristics not made explicit here. In (1.21), commodities are consumed jointly rather than separated into two bundles \( q_1 \) and \( q_2 \). Such separation is clearly possible in principle, but since \( q_1 \) and \( q_2 \), unlike the individual labor supplies, are usually unobservable, there is little point in doing so. Equation (1.21) is maximized subject to the budget constraint

\[
pq + \omega q_1 + \omega^2 q_2 = \mu + \omega^1 T^1 + \omega^2 T^2 \]  

(1.22)

where \( \omega^1 \) and \( \omega^2 \) are the two wage rates and \( T^1 \) and \( T^2 \) the time endowments. Once again, we have a "standard" problem and it has standard solutions with, for example, the two labor supplies determined as a function of family full income, \( \mu + \omega^1 T^1 + \omega^2 T^2 \), price \( p \) and wage rates \( \omega^1 \) and \( \omega^2 \). Since \( q_1 \) and \( q_2 \) are two goods like any other, we can have substitutability or complementarity relationships between them in the usual way. Husband's and wife's leisure being substitutable in looking after children or complementary in holidaying are
4. Simple neoclassical model of labor supply

4. Extensions to the basic model
4.2 Consumption function and durable goods

Many of the most difficult and important choices made by consumers involve decisions relating to the timing of purchases. In a single, timeless, period in which total expenditure is given, such problems are assumed away. In reality, expenditure today may be at the expense of expenditure tomorrow or at some unspecified time in the future, while expenditure on durable goods today will alter the conditions under which future choice is made. This section takes up these problems, again within a largely neoclassical framework. The exposition is mostly in terms of a simple, two-period model although the extensions to many periods will be indicated when appropriate.

In principle, a full neoclassical analysis would model not only the structure of demand over time but also the structure of labor supply, and the interactions of the two might well be important. For example, changes in the anticipated rate of inflation or the yield on assets may well influence the timing of a retirement decision. We shall examine these interactions in more detail in Chapter 12, but for the moment we shall follow most of the literature on the consumption function and take present and expected future income as given, on the supposition that individuals are largely unable to control the number of hours they work or, indeed, whether they work at all. We shall follow this precedent, largely because the analysis is then simpler and more familiar. It should, however, be emphasized that the two approaches yield quite different results; with labor supply exogenous to the consumer, consumption is a function of (among other things) current and expected future income, while with labor supply endogenous, the corresponding wage rates take their place. This implies, for example, that it makes little sense to build macroeconomic models in which the consumption function is defined on labor income taken as exogenous for consumers and in which the labor market is continuously in equilibrium with employment always equal to labor supply. Consumption functions of this type are characteristic of macroeconomic disequilibrium. Labor supply remains important in such contexts, but its role is quite different from that in the equilibrium setting.

Once again, the material discussed next is a prerequisite for Part Four, Chapters 12 and 13.

The budget constraint and preferences

Again, we begin with the simplest possible case. There is one nondurable consumption good, of which \( q_1 \) is consumed in period 1 and \( q_2 \) in period 2. The corresponding prices are \( p_1 \) and \( p_2 \), and incomes (paid at the beginning of each period) are \( y_1 \) and \( y_2 \). The \( p, q \) notation may invite confusion with the