# Measuring Intergenerational Mobility

## ECON 85600

Miles Corak

PhD Program in Economics and Stone Center on Socio-Economic Inequality
The Graduate Center, City University of New York

milescorak.com

Lecture 5

# The workhorse statistical framework
## is based on regression to the mean

$$lnY_{i,t} = \alpha + \beta lnY_{i,t-1} + \epsilon_{i,t}$$

The elasticity between the career adult earnings of a child and that of his or her parents

- indicating the degree of relative earnings mobility across the generations
- cross-country comparisons are often limited to father - son earnings
- a limited sense of absolute differences, and no sense of directional changes, or the possibility of non-linearities

# The workhorse statistical framework

## what about daughters?

The great bulk of the literature is focused on fathers and sons, so cross-national comparisons are pretty well restricted to this group

- there are analyses of daughters
  - generally the elasticity is about the same, or a bit lower
  - but this is more challenging to estimate because lower participation rates and assortative mating may mean that earnings are not a clear indicator of socio-economic status or command over resources
    - how to deal with women who have no earnings?
    - how to recognize the role of assortative mating and the earnings of the partner?
    - family income is increasingly the subject of analysis
    - Chadwick and Solon (2002) *AER* offer one framework

# The usefulness of the intergenerational elasticity

## one way to think about it is as the intergenerational analogue of the GINI coefficient

- falls naturally out of some established theories of the intergenerational process
- an overall summary statistic, indicating the degree to which inequality is passed on across generations

$$Y_{i,t} = e^{\alpha} e^{\beta \ln Y_{i,t-1}}$$

- implying that the ratio of earnings for children from high income ($H$) to low income ($L$) families is:

$$\frac{Y_{H,t}}{Y_{L,t}} = \left(\frac{Y_{H,t-1}}{Y_{L,t-1}}\right)^{\beta}$$

# The usefulness of the intergenerational elasticity

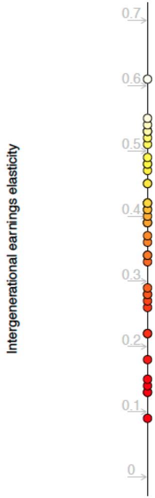
## $\beta$ indicates the degree to which inequality is passed across generations

- according to one estimate, in the US households with children in the top quintile earn 12 times as much as those in the bottom quintile
- the economic advantage children from high income households will have over those from low income households depends upon $\beta$

| $\beta$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| Income advantage | 1.0 | 1.6 | 2.7 | 4.4 | 7.3 | 12 |

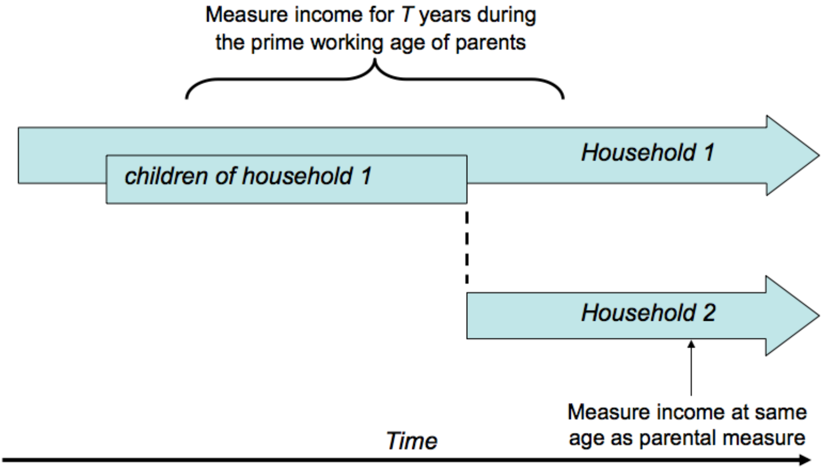
# There is little that can be learned from the literature

## either for the US or for cross-country comparisons



# The imagined ideal source of information

## involves longitudinally tracking household members and their children into adulthood



# Solon suggests the elasticity has been understated

## because of measurement error in the Right Hand Side variable

| Year of father's log earnings | Measure of father's log earnings | | | | |
|---|---|---|---|---|---|
| | Single-year measure | Two-year average | Three-year average | Four-year average | Five-year average |
| 1967 | 0.386<br>(0.079)<br>[322] | | | | |
| | | 0.425<br>(0.090)<br>[313] | | | |
| 1968 | 0.271<br>(0.074)<br>[326] | | 0.408<br>(0.087)<br>[309] | | |
| | | 0.365<br>(0.081)<br>[317] | | 0.413<br>(0.088)<br>[301] | |
| 1969 | 0.326<br>(0.073)<br>[320] | | 0.369<br>(0.083)<br>[309] | | 0.413<br>(0.093)<br>[290] |
| | | 0.342<br>(0.078)<br>[312] | | 0.357<br>(0.088)<br>[298] | |
| 1970 | 0.285<br>(0.073)<br>[318] | | 0.336<br>(0.084)<br>[301] | | |
| | | 0.290<br>(0.082)<br>[303] | | | |
| 1971 | 0.247<br>(0.073)<br>[307] | | | | |

*Notes:* Standard-error estimates are in parentheses, and sample sizes are in brackets.

Source: Gary Solon (1992). "Intergenerational Income Mobility in the United States," *AER* Table 2.

# Attenuation bias

## observed income is only a proxy for lifetime income

$$lnY_{i,t} = \alpha + \beta lnY_{i,t-1} + \epsilon_{i,t}$$

express our model in deviations from means, and use lower case to refer to natural logarithms and to distinguish the generations

$$y_i = \beta x_i + \epsilon_i$$

- an accurate estimate of lifetime incomes place a big demand on our data, and we usually measure them with some noise
- $\epsilon_i$ has the classical properties and in particular is not correlated with $x_i$, but $y_{i,t} = y_i + \upsilon_{it}$ is our proxy for $y_i$, so we are forced to estimate

$$y_{i,t} = \beta x_i + (\epsilon_i + \upsilon_{i,t})$$

the least squares estimate remains consistent, from this perspective measurement error in the child's income raises no special concerns

# Attenuation bias

## observed income is only a proxy for lifetime income

On the other hand, measurement error in parental income does raise a concern, leading to attenuation bias

- if $x_{i,t} = x_i + \upsilon_{it}$ is our proxy for $x_i$, so we are forced to estimate

$$y_i = \beta x_{i,t} + (\epsilon_i + \upsilon_{i,t})$$

$$\text{so that } plim\hat{\beta} = \frac{cov(x_{i,t}, y_i)}{var(x_{i,t})} rr_{x,t},$$

$$\text{where } rr_{x,t} = \frac{var(x_{i,t})}{var(x_{i,t}) + var(\upsilon_{i,t})} \text{ is the "reliability ratio"}$$

- least squares is not consistent, and will be understated if there is noise in the right hand side variable

This is what Solon (1992) was addressing in taking multi-year averages of parental income, or what he and Zimmerman (1992) were after in using IV

# Attenuation bias

will be exacerbated if our data are not representative of the underlying population

There are actually two limitations of our data that could interact to make things even worse. If the data are not drawn from a nationally representative population, then the attenuation bias due to measurement error will be exacerbated

- express $plim\hat{\beta} = \frac{cov(x_{i,t}, y_i)}{var(x_{i,t})} rr_{x,t}$ as $\beta = \widetilde{\beta}\left[1 + \frac{var(\upsilon_{i,t})}{var(x_{i,t})}\right]$
- where $\beta = \frac{cov(x_{i,t}, y_i)}{var(x_{i,t})}$ is the true estimate, and $\widetilde{\beta}$ is the least squares estimate from the noisy data

if our data understates $var(x_{i,t})$, then the attenuation bias from measurement error is made worse

# Life cycle bias

## the relationship between current and life time income changes over the life cycle

The text book case of errors in variables makes an assumption that the slope coefficient between the true and the noisy measure of income is one, but in our application this will not in general be the case.

- our intuition about the evolution of labour market earnings over the life cycle suggests that we should think more generally

$$y_{i,t} = \lambda_t y_i + v_{it}$$

- where $\lambda_t$ may or may not be equal to one, and likely varies over the life cycle
- differential patterns in human capital investment across the population will imply differences in earnings growth

This is not about measurement error, and averaging is not the solution because it reflects differences in variances

# Life cycle bias, some intuition

## the relationship between current and life time income changes over the life cycle

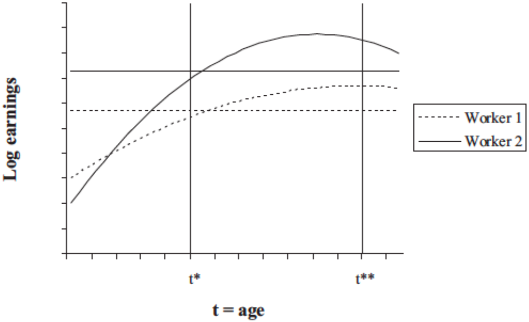



FIGURE 1. ILLUSTRATIVE EXAMPLE OF LOG ANNUAL EARNINGS AND LOG ANNUITIZED LIFETIME EARNINGS

*Note:* For each worker, the upward-sloping line depicts log annual earnings by age, and the horizontal line depicts log annuitized lifetime earnings.

Source: Haider and Solon (2006). "Life Cycle Variation ..." *AER*, Table 2.

# Life cycle bias

## raises issues for both the left and right hand side variables

Unlike measurement error, least square estimates of the intergenerational elasticity are sensitive to life cycle issues for the child's income.

- abstract from measurement error and use $x_i$
- if $y_{i,t} = \lambda_t y_i + \upsilon_{it}$ then $y_{i,t} = \lambda_t \beta x_i + (\lambda_t \epsilon_i + \upsilon_{it})$
- where $plim \hat{\beta} = \lambda_t \beta$
- least squares is inconsistent, and the extent and nature of the bias depends upon the value of $\lambda_t$

The age of the child when income is measured matters.

# Life cycle bias

## raises issues for both the left and right hand side variables

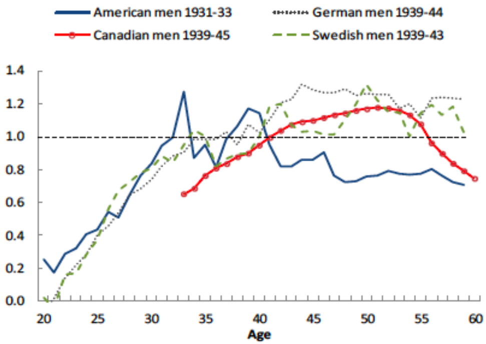
The nature of the bias due to measurement error is more complicated when it is compounded with life cycle bias

- if $x_{i,t} = \lambda_t x_i + \upsilon_{it}$ we have $plim\hat{\beta} = \frac{cov(x_{i,t},y_i)}{var(x_{i,t})} rr_{x,t}$,
- where $rr_{x,t} = \frac{\lambda_t var(x_{i,t})}{\lambda_t^2 var(x_{i,t}) + var(\upsilon_{i,t})}$
- if $\lambda_t < 1$ and pretty small $rr_{x,t}$ could be greater than one

The age of the parent when income is measured matters.

# Cross country estimates of life cycle bias

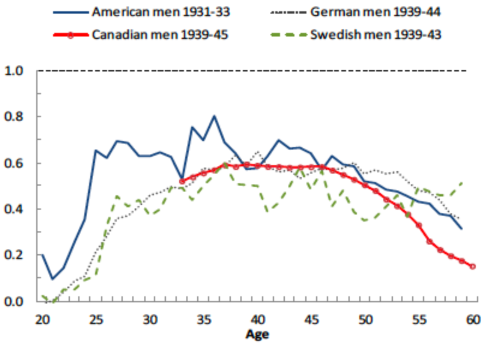
## $\lambda$ seems to follow an inverted U, taking a value of 1 between ages 35 and 40




Source: Chen, Ostrovsky, Piraino (2015). "Intergenerational Mobility ..." Figure 3a.

# Cross country estimates of attenuation bias

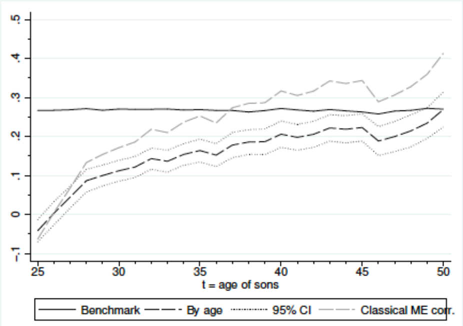
$rr$ also follows an inverted U, reaching 0.6 at age 40 when $\lambda$ is about 1




Source: Chen, Ostrovsky, Piraino (2015). “Intergenerational Mobility ...” Figure 3b.

# An accurate estimate of $\beta$

requires both measurement and life cycle biases to be addressed




Source: Nybom and Stuhler (2015). "Standard Measures ..." Figure 2a.

# 1. The linear correlation coefficient

## mobility in terms of standardized incomes

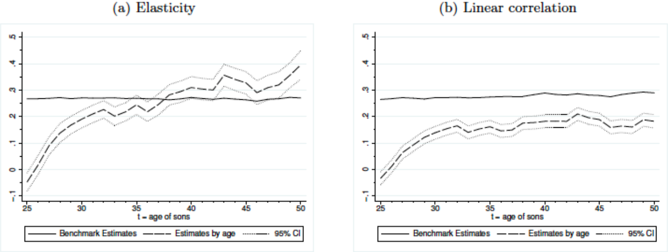
$$\rho(x_i, y_i) = \frac{cov(x_i, y_i)}{\sqrt{var(x_i)}\sqrt{var(y_i)}}$$

this statistic refers to mobility in standardized incomes, controlling for the changes in variance over the generations, note that $\beta = \rho \frac{\sqrt{var(y_i)}}{\sqrt{var(x_i)}}$

- $\rho(x_{i,t}, y_{i,t}) = \rho(x_i, y_i)\sqrt{rr_x rr_y}$
- in contrast to the elasticity, the Pearson correlation coefficient suffer from attenuation bias in both the left hand and right hand side variables
- $\rho$ will be more attenuated than $\beta$ when only measurement error in parental income is corrected

# 1. The linear correlation coefficient

is more attenuated than the elasticity when only measurement error in parental income is corrected




Source: Nybom and Stuhler (2015). "Standard Measures ..." Figure 1a, 1b.

# 2. The rank correlation

mobility in terms of ranks in the respective distributions

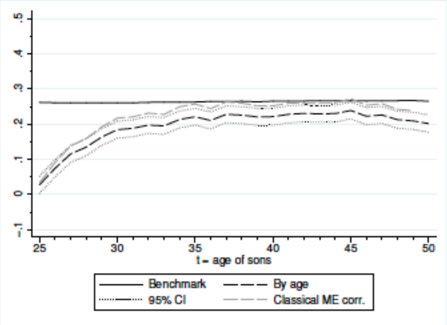
if the ranks of children and parents in their respective distributions are $X$ and $Y$, and they are measured with error so that we observe $\tilde{X} = X + \tilde{u}$ and $\tilde{Y} = Y + \tilde{v}$, then the Spearman rank correlation is

$$\rho^S(X, Y) = \frac{cov(X,Y)}{\sqrt{var(X)}\sqrt{var(Y)}}$$

- no presumption of linearity
- the variances in observed and true ranks are equal by definition, but random errors will generate non-classical errors in ranks because of floor and ceiling effects
- the Spearman correlation like the Pearson is also subject to attenuation bias in errors of both parent and child incomes

# 2. The rank correlation

## suffers from measurement errors, but is more robust




Source: Nybom and Stuhler (2015). "Standard Measures ..." Figure 2c.

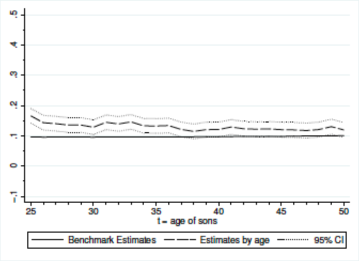
# 3. The transition matrix

The joint distribution of parent and child incomes is represented as transition matrices for some defined quantiles: quartiles, quintiles, even percentiles if the data permit.

- there is particular interest in transitions from the extremes of the income distribution
  - upward mobility from the bottom ... so called “rags to riches” or “intergenerational cycles of poverty”
  - downward mobility from the top ... are there “glass floors”?
- these are the very points in the transition matrix that may be difficult to estimate because at the bottom movement is bounded from below, and at the top from above
  - movement from the bottom to the top, for example, will be overestimated

# 3. The transition matrix

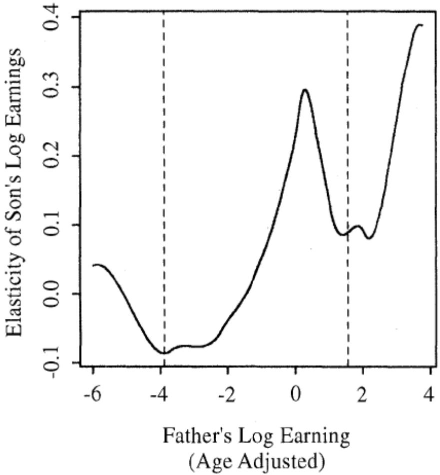
probability of moving from the bottom to the top quintile is overstated




Source: Nybom and Stuhler (2015). “Standard Measures ...” Figure 2d.

# A remaining puzzle

## ‘Peculiar’ nonlinearities in $\beta$ (in Canadian—but also US—administrative data)




Source: Corak and Heisz (1999). *JHR*,Figure 3b.

- Chadwick, Laura and Gary Solon. 2002. "intergenerational Income Mobility Among Daughters." *The American Economic Review*. 92(1): 335-344.
- Chen, Wen-Hao, Patrizio Piraino, and Yuri Ostrovsky. 2015. "Lifecycle Variation, Errors-in-Variables Bias and Nonlinearities in Intergenerational Income Transmission: New Evidence from Canada." Unpublished.
- Corak, Miles. 2006. "Do poor children become poor adults? Lessons from a cross country comparison of generational earnings mobility." In John Creedy and Guyonne Kalb (editors). *Research on Economic Inequality*, Vol. 13. Amsterdam: Elsevier.
- Corak, Miles and Andrew Heisz. 1999. "The Intergenerational Earnings and Income Mobility of Canadian Men: Evidence from Longitudinal Income Tax Data." *Journal of Human Resources*. 34:504-33.
- Corak, Miles, Matthew J. Lindquist and Bhashkar Mazumder. 2014. "A comparison of upward and downward intergenerational mobility in Canada, Sweden and the United States. " *Labour Economics*. 30:185-200.
- Grawe, Nathan. 2006. "The extent of lifecycle bias in estimates of intergenerational earnings persistence. " *Labour Economics*. 13(5): 551–570.
- Haider, Steven, and Gary Solon. 2006. "Life-Cycle Variation in the Association between Current and Lifetime Earnings." *The American Economic Review* 96 (4): 1308–20.

- Nybom, Martin and Jan Stuhler. 2015. “Biases in Standard Measures of Intergenerational Income Dependence. ” Unpublished manuscript.
- Solon, Gary. 1992. “Intergenerational Income Mobility in the United States.” *The American Economic Review* 82 (3): 393–408.
- Zimmerman, David J. 1992. “Regression Toward Mediocrity in Economic Stature.” *The American Economic Review* 82 (3): 409–29.