

# Intergenerational Mobility in Theory

## ECON 85600

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# Reflecting on the last lecture

## and the challenges for theory

- ① A wide range of estimates of the intergenerational earnings elasticity
  - we certainly need to be aware of a host of data limitations and measurement issues
- ② There are substantive differences across space
  - between countries
  - within countries (which are highlighted in supplemental readings)
- ③ We might also wonder if the elasticity differs over time within a country
- ④ And the elasticity also seems to differ, in peculiar ways, across the income distribution

*“... practically all the advantages or disadvantages of ancestors tend to disappear in only three generations: “from shirtsleeves to shirtsleeves in three generations.” Parents in such “open” societies have little effect on the earnings of grandchildren and later descendants. Therefore, they have little incentive to try to affect the earnings of descendants through family reputation and other means.”*

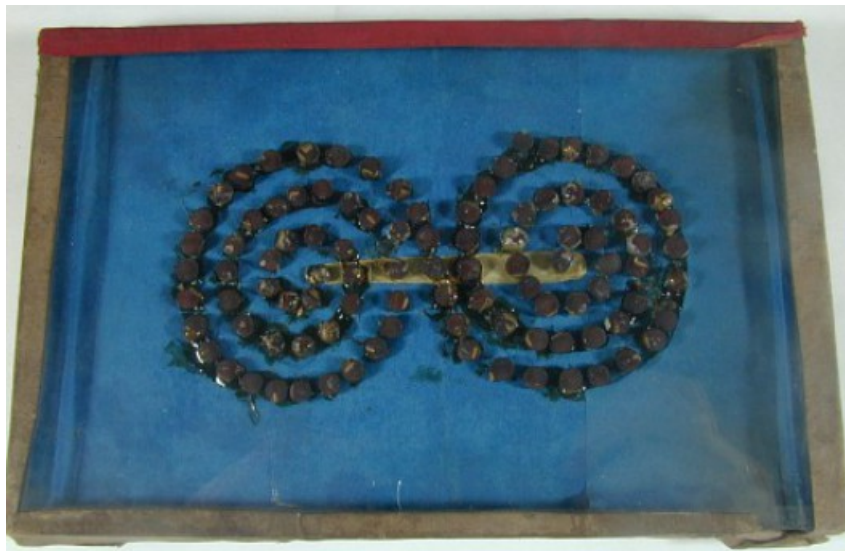
— Gary Becker and Nigel Tomes, *Journal of Labor Economics*, 1986, p. S28.

*“... all social mobility is governed by a simple underlying law, independent of social structure and government policy:*

$$x_{t+1} = bx_t + e_t$$

*where  $x_t$  is the underlying social status of a family in generation  $t$ ,  $e_t$  is a random component, and  $b$  is in the region 0.7-0.8. This simple law of mobility makes surprising predictions about the earlier history of social elites and underclasses observed at any time....[It] implies that on average, the status of the descendants will move toward the mean for the society generation by generation. When the persistence rate,  $b$ , is as high as 0.8, this is a slow process, taking many hundreds of years for families who are initially far above or below the mean.”*

— Gregory Clark, *The Son Also Rises*, 2014, p. 212



In the late 1880s Sir Francis Galton planted a garden he asked his friends to help him plant sweet peas

TABLE 2.

## PARENT SEEDS AND THEIR PRODUCE.

The proportionate number of sweet peas of different sizes, produced by parent seeds also of different sizes, are given below. The measurements are those of their mean diameters, in hundredths of an inch.

Diameter of Parent Seed.	Diameters of Filial Seeds.								Total.	Mean Diameter of Filial Seeds.	
	Under 15.	15-	16-	17-	18-	19-	20-	Above 21-		Observed	Smoothed
21	22	8	10	18	21	13	6	2	100	17.5	17.3
20	23	10	12	17	20	13	3	2	100	17.3	17.0
19	35	16	12	13	11	10	2	1	100	16.0	16.6
18	34	12	13	17	16	6	2	0	100	16.3	16.3
17	37	16	13	16	13	4	1	0	100	15.6	16.0
16	34	15	18	16	13	3	1	0	100	16.0	15.7
15	46	14	9	11	14	4	2	0	100	15.8	15.4

# The set-up

- An overlapping generations model in which all individuals live for two periods
  - they spend their first period as a child, and their second period as a parent
  - there is one parent in the family, who has one child
    - abstracts from assortative mating and the role of the marriage market
    - in period one, the parent divides current income between consumption and investment in the child's earnings capacity
    - in period two, the child receives income from the market, and in turn makes a consumption and investment decision

# The set-up

- An overlapping generations model in which all individuals live for two periods
- The child also receives an “endowment” from the parent independent of investment choices
  - the endowment independently influences earnings
  - it also influences the productivity of investments
    - which are subject to a diminishing marginal productivity

Becker and Tomes, The Intuition

# The set-up when capital markets are perfect

“Access to capital markets ... separates the transmission of earnings from the generosity and resources of parents.”

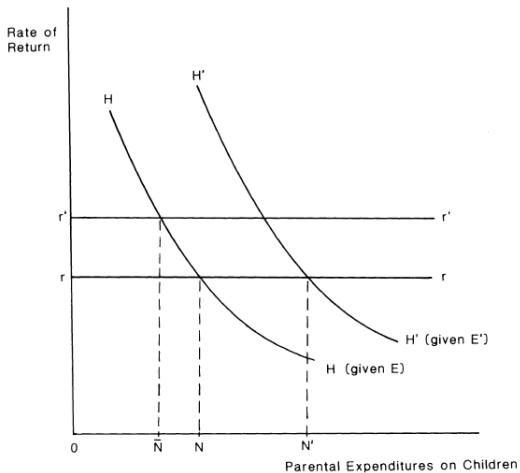


FIG. 1.—Rates of return on parental expenditures on children



Becker and Tomes, The Intuition

# The set-up when capital markets are not perfect

“Earnings of children now depend directly on the earnings of parents as well as indirectly through the transmission of endowments.”

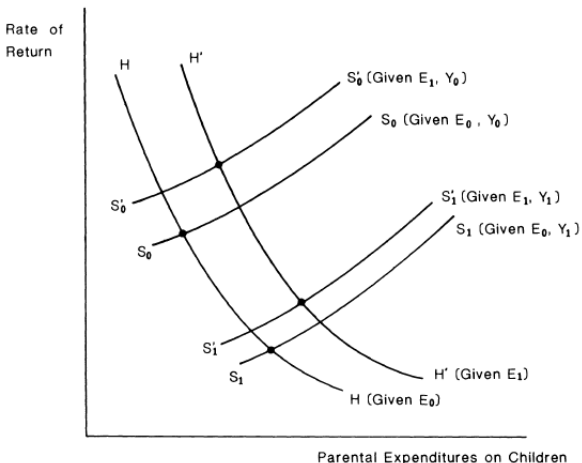


FIG. 2.—Parental expenditures on children, with capital constraints

A simplified Becker-Tomes model

# The inheritability of endowments is one key parameter determining the degree of intergenerational mobility, but there can be others

Let family  $i$  contains one parent in generation  $t-1$  and one child in generation  $t$

- the child's lifetime earnings is determined by the amount of human capital, which has a rate of return of  $\rho$

$$\ln y_{i,t} = \mu + \rho h_{i,t} \quad (1)$$

- the parent makes an investment of  $l_{i,t-1}$  in the human capital of the child, which is given as:

$$h_{i,t} = \theta \ln l_{i,t-1} + e_{i,t} \quad (2)$$

- where  $\theta \geq 0$ , and where  $e_{i,t}$  is a human capital endowment that does not depend upon parental investments and is “inherited” from the previous generation:

$$e_{i,t} = \delta + \lambda e_{i,t-1} + v_{i,t} \quad (3)$$

# The inheritability of endowments is one key parameter determining the degree of intergenerational mobility, but there can be others

With respect to equation (3) Becker and Tomes say:

*Some children have an advantage because they are born into families with substantial ability, a strong emphasis on childhood learning, and other favorable cultural and genetic attributes. Both biology and culture are transmitted from parents to children, one encoded in the DNA and the other in a family's heritage. Much less is known about the transmission of cultural attributes than of biological ones, and even less is known about the relative contributions of biology and culture to the distinctive endowment of each family. We do not need to separate cultural from genetic endowments, and we will not try to specify the exact mechanism of cultural transmission.*

*(Becker and Tomes, 1986, pages S4-S5).*

A simplified Becker-Tomes model

# The inheritability of endowments is one key parameter determining the degree of intergenerational mobility, but there can be others

The child's lifetime earnings depend upon parental investments, but how does the parent decide upon how much to invest?

$$\ln y_{i,t} = \mu + \rho\theta \ln l_{i,t-1} + \rho e_{i,t} \quad (4)$$

- the family must allocate parental lifetime earnings  $y_{i,t-1}$  between the parent's own consumption  $C_{i,t-1}$  and an investment  $l_{i,t-1}$  in the child's human capital. The budget constraint it faces is:

$$y_{i,t-1} = C_{i,t-1} + l_{i,t-1} \quad (5)$$

- and it maximizes a Cobb-Douglas utility function

$$U_i = (1 - \alpha) \ln C_{i,t-1} + \alpha \ln y_{i,t} \quad (6)$$

- where  $0 < \alpha < 1$  represents the degree of parental altruism

A simplified Becker-Tomes model

# The inheritability of endowments is one key parameter determining the degree of intergenerational mobility, but there can be others

The first order conditions can be solved for the optimal investment in the child's human capital

$$l_{i,t-1} = \left\{ \frac{\alpha\rho\theta}{1 - \alpha(1 - \rho\theta)} \right\} y_{i,t-1} \quad (7)$$

- the amount of investment in the child's human capital will be higher if
  - the parent has higher income
  - the parent is more altruistic
  - the return to human capital is higher

A simplified Becker-Tomes model

# The inheritability of endowments is one key parameter determining the degree of intergenerational mobility, but there can be others

If this is the optimal investment, then the child's earnings are:

$$\ln y_{i,t} = \mu^* + \rho\theta \ln y_{i,t-1} + \rho e_{i,t} \quad (8)$$

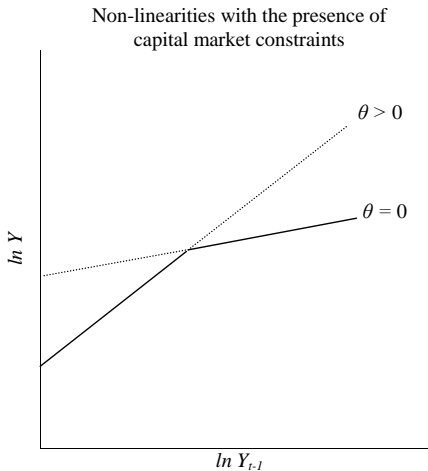
- where  $\mu^* = \mu + \rho\theta \ln \left( \frac{\alpha\rho\theta}{1-\alpha(1-\rho\theta)} \right)$
- this looks like our workhorse regression to the mean model, but the error is not well behaved
- it is correlated with  $\ln y_{i,t-1}$  because both depend upon  $e_{i,t-1}$
- it is a first-order autoregression with serially correlated error following a first-order autoregression
- in a steady state the slope coefficient, what we call the intergenerational elasticity, is the sum of the two autoregressive parameters divided by 1 plus their product

$$\beta = \frac{\rho\theta + \lambda}{1 + \rho\theta\lambda} \quad (9)$$

Some predictions

# Non linearities

the model predicts a non-linear, concave, pattern in the elasticity



Some predictions

# Inequality

in the cross section is determined by the same factors as mobility, but also another

In the steady state there will be a relationship between intergenerational mobility and cross-sectional inequality

- in the steady state a first-order autoregression with a first-order autoregressive error can be expressed as a second-order autoregression with a white noise error
- equation (8) can be expressed as

$$\ln y_{i,t} = (1 - \lambda)(\mu^* + \rho\delta) + [\rho\theta + \lambda] \ln y_{i,t-1} - \rho\theta\lambda \ln y_{i,t-2} + \rho v_{i,t} \quad (10)$$

- the variance of a variable following a second-order autoregression is

$$\text{Var}(\ln y_{i,t}) = \frac{[1 + \rho\theta\lambda] \rho^2}{[1 - \rho\theta\lambda] (1 - \lambda^2) [1 - (\rho\theta)^2]} \text{Var}(v_{i,t}) \quad (11)$$



# Geometric declines in mobility?

even if  $\beta$  is pretty large it will diminish quickly over successive generations

- many analysts presume that the degree of mobility will decline geometrically
- even a rather high intergenerational elasticity between two generations will rapidly fall off over multiple generations
- is  $\beta^2$  the right to think about the link between a child and a grandparent's earnings, and  $\beta^3$  between children and great-grandparents?
- equation (10) suggests not

# Mobility across multiple generations

$$\ln y_{i,t} = (1 - \lambda)(\mu^* + \rho\delta) + [\rho\theta + \lambda] \ln y_{i,t-1} - \rho\theta\lambda \ln y_{i,t-2} + \rho v_{i,t}$$

- this seems to imply that higher grandparent earnings reduce child earnings, and an even more rapid regression than geometric
- we should not interpret this to mean that an exogenous increase in grandparent earnings is of detriment to the grandchild
  - the negative relationship is conditional on parental earnings
  - if the parent is not making more when grandparent income is higher it means the parent's endowment is low, and it is this bad draw that is getting passed on to the child
  - this is a testable implication of the model, and to the extent that there is a literature on this, the finding is that grandparent earnings figure positively

# Mobility across multiple generations

There is a need for more refined modeling of multiple generations

- ① grandparents as well as parents could invest directly in the child's human capital
- ② grandparents can be directly implicated in the child's lives, living in the same household and contributing to its "culture"
- ③ genes may skip generations
- ④ "group effects" may matter and these will be correlated with the status of both parents and grandparents
- ⑤ measurement error may create the appearance of grandparent effects

# Mobility across multiple generations

As an exercise, reconsider the model by letting endowments be transmitted as

$$e_{i,t} = \delta + \lambda_1 e_{i,t-1} + \lambda_2 e_{i,t-2} + v_{i,t} \quad (12)$$

where  $0 < \lambda_2 < \lambda_1 < 1$

- what are the implications for the child's income? Is it possible for the grandparent earnings to enter positively?
- how should this be interpreted?

## Social investment

a child's human capital may reflect public investments

Let the parent's budget constraint be:

$$(1 - \tau)y_{i,t-1} = C_{i,t-1} + I_{i,t-1} \quad (13)$$

Where  $\tau$  is a proportional tax and the family's problem concerns the allocation of after tax income

Let the human capital of the child be determined both by private and public investment

$$h_{i,t} = \theta \ln [I_{i,t-1} + G_{i,t-1}] + e_{i,t} \quad (14)$$

Where  $G_{i,t-1}$  is government investment in the child's education, and where the degree of progressivity in the tax system is represented by these expenditures

# Social investment

increases the degree of social mobility if it is “progressive”

In this model the intergenerational elasticity can be solved for as in Solon (2004) to yield:

$$\beta = \frac{(1 - \gamma)\rho\theta + \lambda}{1 + (1 - \gamma)\rho\theta\lambda} \quad (15)$$

where  $\gamma > 0$  is an indicator of how “progressive” public investments are, with a positive value indicating that the ratio of public investment to parental after-tax income falls with parental income.

# Readdressing our puzzles

theory has informed us about the host of parameters driving differences across time and space

$$\ln Y_{i,t} = \alpha + \beta \ln Y_{i,t-1} + \epsilon_{i,t}$$

The elasticity between the career adult earnings of a child and that of his or her parents is an amalgam of structural factors

- reflecting differences in the inheritability of endowments, access to capital markets, returns to education, the progressivity of public investments
- cross-country comparisons require us to identify the causal role of these factors, as do changes over time
- we still have a very limited sense of non-linearities and what is driving them

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