Labour Supply on the Extensive Margin

The Participation Decision

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Lecture 3
Labor Economics I
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Introduction

The labour force participation decision occurs at $H = 0$ for a labour force non-participant, that is to say, $MRS_{XL} > W/P$ at $y$

- the subjective rate of exchange of $X$ and $L$ exceeds the market rate of exchange
- the value of an additional hour of non-market time exceeds the value of commodities resulting from an additional hour of work.
- the labour force participant’s utility maximization is an interior solution
- at $H = 0$ the $MRS_{XL} \leq W/P$ implying that utility is higher when some time is spent working

We define a binary variable that summarizes this

$$P_i = 0 \text{ iff } H_i = 0, \text{ that is } MRS_{XL} > W/P \text{ at } H_i = 0$$

$$P_i = 1 \text{ iff } H_i > 0, \text{ that is } MRS_{XL} < W/P \text{ at } H_i = 0$$
Definition of the Reservation wage

\[ w_i^R \equiv MRS_{XL}^i(H_i, w_i, y_i) \]

The reservation wage is the \( MRS_{XL} \) subject to the full income budget constraint

- the location of the budget constraint and the utility function determine \( w_i^R \)
Definition of the Reservation wage

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- the location of the budget constraint and the utility function determine \( w_i^R \)
- the value of of \( w_i^R \) at the endpoint of the budget constraint is the only value relevant for the participation decision

\[ w_i^0 = MRS_{XL}^i(H_i = 0, w_i, y_i) \]
\[ W_i^0 = P \times w_i^0 \]
\[ = P \times MRS_{XL}^i(H_i = 0, w_i, y_i) \]
Definition of the Reservation wage

\[ w^R_i \equiv MRS^i_{XL}(H_i, w_i, y_i) \]

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\[ W^0_i = P \times w^0_i \]
\[ = P \times MRS^i_{XL}(H_i = 0, w_i, y_i) \]

- because of diminishing margin rates of substitution \( w^R_i \) and \( W^R_i \) are positive functions of \( H_i \)
The market wage and the reservation wage

The relationship between \( w \) and \( w^R \) is the essence of the participation decision.

Consider an increase in the market wage:

- the participation response to such a change depends upon the magnitudes of the change in \( w \) and the initial position of the individual.
The market wage and the reservation wage

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  - If the individual is a labour force participant, the hours of work adjust but participation status is unchanged.
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- The participation response to such a change depends upon the magnitudes of the change in $w$ and the initial position of the individual:
  - If the individual is a labour force participant, the hours of work adjust but participation status is unchanged.
  - If the individual is a labour force non-participant there are two possible responses:
    - No response.
    - Entry into the labour force.
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    1. No response.
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  - If the individual is a labour force participant, the hours of work adjust but participation status is unchanged.
  - If the individual is a labour force non-participant there are two possible responses:
    1. No response
    2. Entry into the labour force
The Labour Force Participation of a group

Ben-Porath (1973) develops a probabilistic framework helpful in predicting how the Labour Force Participation Rate (LFPR) changes for changes in parameters. It applies to groups of individuals, and two polar cases are examined, both of which predict $\frac{\delta LFPR}{\delta W} > 0$

- Model A assumes preferences differ between individuals but the market wage is identical
- Model B assumes the market wage differs between individuals but they all have identical preferences

Focus on Model A in what follows, and leave the examination of Model B for individual work.
The Labour Force Participation of a group

Model A: variable preferences, constant market wage rates

A case in which tastes differ among individuals in the group (so that reservation wages differ), but market wage rates are identical for all members of the group.

1. $W_i^R = W_i^R$ varies across group members, and is characterized by a probability distribution function $g(W^R)

2. $W_i = W$, market wage rates are the same for all members of the group.

Under these circumstances the group LFPR may be defined by assuming a particular form for the probability distribution function.

- participation occurs if $W_i > W_i^R$. This inequality may be represented probabilistically as the area under the probability distribution function between 0 and $W$. 
The Labour Force Participation of a group
Model A: variable preferences, fixed market wage rate

Probability Distribution Function: Proposition 1

Reservation Wage Distribution

$\mu^1$
The Labour Force Participation of a group
Model A: variable preferences, fixed market wage rate

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\[ \mu^1 \]
The Labour Force Participation of a group
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Cummulative Distribution Function: Proposition 1

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Reservation Wage Distribution

θ^1
µ^1

Reservation Wage Distribution
The Labour Force Participation of a group
Model A: variable preferences, fixed market wage rate

Cumulative Distribution Function: Proposition 1
Reservation Wage Distribution

\[ \theta^w \]
\[ \theta^1 \]
\[ W \]
\[ \mu^1 \]
The Labour Force Participation of a group
Model A: variable preferences, fixed market wage rate

Probability Distribution Function: Proposition 1
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Cummulative Distribution Function: Proposition 1
Reservation Wage Distribution

0
A
A+B
1
W µ 1

Reservation Wage Distribution
The Labour Force Participation of a group

Model A: variable preferences, fixed market wage rate

\[ \theta_g \equiv LFPR_g = \int_0^W g(W) dW = G(W) \]

An immediate prediction is that \( LFPR_g \) changes in the same direction as \( W \)

\[ \theta_1(W = W_1) = \int_0^{W_1} g(W) dW = G(W_1) = A \]

\[ \theta_2(W = W_2) = \int_0^{W_2} g(W) dW = G(W_2) = A + B \]

The fact that \( A + B > A \) proves our proposition, another way of just saying \( G(W_2) > G(W_1) \) for \( W_2 > W_1, \theta_2 > \theta_1 \), implying \( \frac{\delta \theta}{\delta W} > 0 \)
Aside on the statistical measurement of labour force concepts

Labour force surveys, like the Current Population Survey in the United States, attempt to measure the faction of the population for which $W^R > W$, the Labour Force Participation Rate.

- These are usually monthly surveys, and usually conducted during the week following the “reference” week, the week containing the 15th day of the month.

- Labour force survey concepts are measured according to respondent activity during the reference week.
The Labour Force Participation of a group

Aside on the statistical measurement of labour force concepts

The determination of labour force status involves placing each individual in the survey into one of three categories:

1. Employed
2. Unemployed
3. Not in the labour force
The Labour Force Participation of a group

Aside on the statistical measurement of labour force concepts

Basic definitions

1. Employed (E)
The Labour Force Participation of a group

Aside on the statistical measurement of labour force concepts

Basic definitions

1. Employed (E)
   - did any work at all at a job or business, that is, paid work in the context of an employer-employee relationship, or self-employment
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   - had a job but were not at work due to factors such as own illness or disability, personal or family responsibilities, vacation, labour dispute or other reasons
   - the Employment Rate is:

$$ ER = E / POP $$
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Basic definitions

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2. Unemployed (U)
The Labour Force Participation of a group

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Basic definitions

1. Employed (E)
2. Unemployed (U)

- on temporary layoff during the reference week with an expectation of recall and were available for work

\[ \text{LF} = E + U \]

\[ \text{Unemployment Rate} = \frac{U}{\text{LF}} \]
The Labour Force Participation of a group

Aside on the statistical measurement of labour force concepts

Basic definitions

1. Employed (E)
2. Unemployed (U)
   - on temporary layoff during the reference week with an expectation of recall and were available for work
   - or were without work, had looked for work in the past four weeks, and were available for work

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\text{LF} = E + U
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The Unemployment Rate is:

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   - together E and U make up the Labour Force: \( LF = E + U \)
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   ▶ together E and U make up the Labour Force: \( LF = E + U \)
   ▶ the Unemployment Rate is: \( UR = U / LF \)
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Basic definitions

1. Employed (E)
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3. Not in the Labour Force (NILF)
The Labour Force Participation of a group

Aside on the statistical measurement of labour force concepts

Basic definitions

1. Employed (E)
2. Unemployed (U)
3. Not in the Labour Force (NILF)
   - unwilling or unable to offer or supply labour services under conditions existing in their labour markets during the reference week

\[ \text{Labour Force Participation Rate (or just Participation Rate)} = \frac{\text{Employed (E)}}{\text{Population (POP)}} \]
The Labour Force Participation of a group

Aside on the statistical measurement of labour force concepts

Basic definitions

1. Employed (E)
2. Unemployed (U)
3. Not in the Labour Force (NILF)
   - unwilling or unable to offer or supply labour services under conditions existing in their labour markets during the reference week
   - that is, they were neither employed nor unemployed

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\text{Labour Force Participation Rate (or just Participation Rate)} = \frac{LF}{POP}
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Aside on the statistical measurement of labour force concepts

Basic definitions

1. Employed (E)
2. Unemployed (U)
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- unwilling or unable to offer or supply labour services under conditions existing in their labour markets during the reference week
- that is, they were neither employed nor unemployed
- the Labour Force Participation Rate (or just Participation Rate) is

\[ LFPR = \frac{LF}{POP} \]
The Labour Force Participation of a group

Model A: variable preferences, fixed market wage rate

Probability Distribution Function: Proposition 2

Reservation Wage Distribution
The Labour Force Participation of a group
Model A: variable preferences, fixed market wage rate

Probability Distribution Function: Proposition 2

Reservation Wage Distribution

W \mu^1
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Probability Distribution Function: Proposition 2

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Model A: variable preferences, fixed market wage rate

Two related propositions

1. An increase in the mean reservation wage implies that LFPR falls, that is \( \frac{\delta \theta}{\delta \mu_{WR}} < 0 \)

2. The closer the market wage is to the mode of the reservation wage distribution, the larger \( \frac{\delta \theta}{\delta W} \) since \( \frac{\delta \theta}{\delta W} = \frac{dG(W)}{dW} = g(W) \)

This latter prediction is consistent with the historical stylized facts, for example consider the substantial growth of the LFPR among women, who had participation rates around 50% a generation or so ago, implying \( W \approx \mu \) was the case.
Summary and things to do

1. Work through Model B in Ben-Porath (1973)
2. Use your understanding of labour supply on the intensive and extensive margin to continue work on the assignment for the next due date, relying on course readings
3. Think about the presentation and term paper topics.